The Term Structure of Bond Liquidity*

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II Introduction

The risk of being unable to sell an asset at its fair value is one of the main risks associated with securities investment. Such liquidity risk is of particular interest for bonds, since they offer investors the opportunity to wait for a bond’s maturity and thereby avoid transaction costs. This option creates a relation between the time until a bond’s maturity and the liquidity premium investors require to invest in the bond. As this relation affects trading strategies, optimal portfolio allocations, price discounts, and capital costs, it is important to all investors and issuers active in global bond markets.

Although there are numerous papers empirically investigating the relation between liquidity premia and maturity, there is little consensus even on the most fundamental question: What is the shape of the term structure of liquidity premia? Empirically, the term
structure is found to be decreasing (Ericsson and Renault (2006)), increasing (Dick-Nielsen, Feldhütter, and Lando (2012)), or U-shaped (Longstaff (2004)). With respect to liquidity transmission between maturity segments, some informal arguments explain empirically observed spillovers, however, we are not aware of any formal equilibrium explanation. Moreover, the literature offers no explanation for our puzzling empirical observation that bonds with very short or long maturities are rarely traded, while there is an active secondary market for bonds with intermediate maturities.

We suggest a parsimonious equilibrium model that explains the seemingly conflicting empirical results on the shape of the term structure of liquidity premia. Our model also provides an investor-based rationale for spillovers from short- to long-term premia (see Goyenko, Subrahmanyam, and Ukhov (2011) for empirical evidence on liquidity transmission for U.S. Treasury bid-ask spreads). In addition, our unified framework explains the empirically observed hump-shaped term structure of trading volume and the well-known aging effect (see, e.g., Warga (1992), Edwards, Harris, and Piwowar (2007)): other things equal, old bonds trade less frequently than newly issued bonds.

In our model, agents with heterogeneous investment horizons trade bonds with a continuum of different maturities in a market with two simple frictions: transaction costs and shocks to investors’ time preference parameter. If a preference shock occurs, the investor faces the trade-off between the cost (in terms of utility) of awaiting the asset’s maturity, which is higher for long-term bonds, and the bid-ask spread charged by an exogenous market maker or dealer. Prior to the preference shock, the investor determines her optimal portfolio allocation by comparing the higher return earned when holding a long-term bond until the maturity date to the higher expected costs of selling this asset in case of a preference shock.

Our model offers five key testable predictions. First, assets with very short maturities are traded less frequently, as are assets with long maturities. The first effect arises because investors have lower disutility from waiting than from paying the bid-ask spread
when maturity is short. As only investors who experience comparatively few preference shocks hold assets with long maturities, these assets are rarely traded as well. Second, since these low preference shock investors still hold a proportion of aged (formerly long-term, but now short-term) bonds, our model endogenously explains the well-documented aging effect. We believe that ours is the first equilibrium model to explain the impact of aging on trading volume via a simple transaction cost friction. Third, liquidity premia in bond yields computed from ask prices are negligible for short maturities, and increase for longer maturities. The increasing term structure arises, even for constant bid-ask spreads, because the disutility from waiting increases with maturity. For longer maturities, the term structure flattens out as investors with low probabilities of preference shocks dominate. Fourth, liquidity premia from bid yields depend on the term structure of bid-ask spreads. If transaction costs do not depend on the bond’s maturity, short-term liquidity premia are large, then decrease and flatten out at longer maturities. If transaction costs are increasing in maturity, the term structure takes on a U-shape. Fifth, investor-specific portfolio decisions lead to a transmission of liquidity shocks from the short end to the long end of the term structure, but not vice versa.

We verify these key model predictions empirically using transaction data for highly rated U.S. corporate bonds from the Trade Reporting and Compliance Engine (TRACE) database. The results of multiple regressions confirm that transaction volume is hump-shaped and bonds are traded less frequently as they age. To calculate the liquidity component in bond yields, we employ two completely different approaches. First, we follow Longstaff (2005) and compute liquidity premia as the difference of bond yields from trade prices and theoretical prices that are computed from a bootstrapped credit risky curve using Treasury yields and credit default swap (CDS) premia. Second, we implement the methodology of Dick-Nielsen et al. (2012) and identify the liquidity component using an indirect, regression-based approach. All analyses as well as multiple robustness checks show that liquidity premia computed from ask prices are monotonically increasing with
a decreasing slope. Liquidity premia computed from bid prices are U-shaped with significant liquidity premia for very short maturities. Finally, a vector autoregression analysis confirms spillovers from short- to long-term liquidity premia.

Our paper adds to several strands of literature. Ericsson and Renault (2006) model the liquidity shock for assets with different maturities as the jump of a Poisson process that forces investors to sell their entire portfolio to the market maker, who charges a proportional spread. Liquidity premia are downward-sloping because only current illiquidity affects asset prices, and because investors have the option to sell assets early to the market maker at favorable conditions. Kempf, Korn, and Uhrig-Homburg (2012) extend this analysis by modeling the intensity of the Poisson process as a mean-reverting process. In this setting, liquidity premia depend on the difference between the average and the current probability of a liquidity shock, and can exhibit a number of different shapes. In contrast to these papers, we allow investors to trade-off the transaction costs when selling immediately versus the disutility from awaiting the bond’s maturity. By endogenizing investors’ trading decisions in bonds of different maturities, our model provides an equilibrium-based explanation for spillovers of liquidity shocks between different ends of the maturity range.

Feldhütter (2012) is most closely related to our study, since he considers an investor’s optimal decision to a holding cost shock. Search costs allow market makers to charge a spread, which results in a difference between the asset’s fundamental value and its bid price. However, Feldhütter (2012) abstracts from aging because in his model, bonds mature randomly with a rate of $\frac{1}{T}$. Additionally, his model cannot accommodate any spillover effects between maturities, as he does not simultaneously consider bonds of different maturities.

Besides supporting the equilibrium model predictions, our results provide an explanation for the variation in the term structures found in previous empirical studies. Studies that document a decreasing term structure (Amihud and Mendelson (1991), Ericsson and Renault (2006)) or a U-shaped term structure (Longstaff (2004)) use mid quotes or ask
quotes net of a spread component such as brokerage costs. In contrast, Dick-Nielsen et al. (2012) find an increasing term structure for the U.S. corporate bond market computed from average quarter-end trade prices. However, trade prices in this market are dominated by buy transactions. Hump-shaped (Koziol and Sauerbier (2007)) or variable term structures (Kempf et al. (2012)) arise from a varying mixture of bid and ask prices. Hence, consistent with our theoretical predictions, the shape of the liquidity term structure is crucially driven by whether most transactions occur at the dealer’s bid or ask price.

Last, our paper contributes to the growing literature on asset pricing in heterogeneous agents models. Like Beber, Driessen, and Tuijp (2012), we study optimal portfolio choice of heterogeneous investors faced with exogenous transaction costs in a stationary equilibrium setting. Duffie, Gârleanu, and Pedersen (2005) and Vayanos and Wang (2007) endogenize transaction costs through search costs and bargaining power. None of these studies, however, can address the relation between maturity and liquidity as they do not simultaneously consider assets with different finite maturities. We show that even when transaction costs are identical for all maturities, liquidity shocks are transmitted from short-term to long-term bonds via heterogeneous investors.

II Model Setup

This section presents an extension of the Amihud and Mendelson (1986) model adapted to bond markets. We present the model setting in Section A, describe the equilibrium in Section B, and provide a discussion of the differences between our model and the one of Amihud and Mendelson (1986) in Section C.
A Setting

In our continuous-time economy with cash as the numeraire, there are two types of assets: the money market account in infinite supply paying a constant nonnegative return \( r \) and a continuum of illiquid zero-coupon bonds with maturity between 0 and \( T_{\text{max}} \) at which they pay one unit of cash. The difference in the yield to maturity between investments in a zero-coupon bond of maturity \( T \) and the money market account gives rise to the liquidity term structure. Bonds are perfectly divisible, but short selling bonds and borrowing via the money market account are not allowed. For each initial maturity \( T_{\text{init}} \) between 0 and \( T_{\text{max}} \), new bonds are issued with a rate \( \frac{1}{T_{\text{init}}} \).\(^1\) Hence, in steady state, bonds of the same initial maturity are equally distributed with respect to their remaining time to maturity.

We consider two types of agents: high-risk investors (type H) and low-risk investors (type L). Each investor is infinitesimally small and we assume that new infinitesimally small investors of each type enter the economy so that total wealth from type-\( i \) investors arrives with rate \( w_i \). Our risk-neutral investors maximize expected lifetime utility, and time-0 utility from future consumption of cash \( c_T \) at time \( T \) is given by \( u_i (c_T) = e^{-\int_0^T \delta_i(t) dt} c_T \), where \( \delta_i(t) \) for \( i \in \{ H, L \} \) is the investor-specific discount rate, which we define below. In addition, we assume that (unmodeled) dealers act as intermediaries for the illiquid bonds: they provide liquidity via bid and ask quotes at which they stand ready to trade. They are compensated for providing immediacy by an exogenous bid-ask spread \( s \in (0, 1) \), i.e., they quote an ask price \( p^{\text{ask}}(T) = p(T) \) and a bid price \( p^{\text{bid}}(T) = (1 - s) \times p(T) \).\(^2\) As the bid-

\(^1\)Note that with the assumption of an issuance rate \( \frac{1}{T_{\text{init}}} \), we assume that the total number of bonds is equally distributed between the initial maturities and maturity dispersion is exogenously determined. This assumption is supported, for example, by firms managing rollover or funding liquidity risk by spreading out the maturity of their debt (Choi, Hackbarth, and Zechner (2016), Norden, Roosenboom, and Wang (2016)).

\(^2\)In contrast to our approach, some recent papers model bid-ask spreads endogenously in a search-based framework. Applying a search model introduces limiting restrictions on the number of different
ask spread is assumed to be exogenous, we determine the equilibrium ask price and derive the bid price directly from it. We consider the case of constant bid-ask spreads, but relax this assumption in Section IV.

Investors choose their portfolio allocation across available assets taking into account that each investor experiences a single preference shock with Poisson rate $\lambda_i$, $i \in \{H, L\}$, $\lambda_L < \lambda_H$, that decreases utility of future consumption by irreversibly changing the discount rate $\delta(t)$ from $r$ to $r + b > r$. Economically, this shock can capture different phenomena. For example, we can interpret this event as a funding liquidity shock (see Brunnermeier and Pedersen (2009)), hedging needs in another market (see Vayanos and Wang (2007)), financing costs (see Duffie et al. (2005)), or any other shock that decreases the bond’s convenience yield or its future value to the investor. In a broader sense, the shock can also be interpreted as a (hypothetical) increase in the investor’s individual risk aversion that increases her discount rate for future payments and leads to a trading incentive as in Guiso, Sapienza, and Zingales (2014) or, for fund managers, as an increased probability of future redemptions.\(^3\) Technically, the shock to the investor’s time-preference rate is similar to a holding cost shock for the bond as in Duffie et al. (2005) or Feldhütter (2012). Note, however, that in these two papers, the shock is reversed after a stochastic period of time. In our setting, as in He and Milbradt (2014), the shock is permanent.\(^4\)

\(^3\)Huang (2015) shows that equity mutual funds sell illiquid securities when expected market volatility increases to protect themselves against investors’ redeeming their funds. He shows that, e.g., mutual funds with high outflow volatility or from small fund families have strong preferences for liquidity (our high-risk investors). In contrast, institutional investors insulated from investor flows (like, e.g., closed-end funds) have long investment horizons and could represent our low-risk investors.

\(^4\)While we assume a permanent shock for mostly technical reasons, our specification also allows for a cleaner interpretation, as uncertainty for the investor is resolved after the shock.
As a reaction to the shock, the investor decides for each bond whether to sell it at the bid price and consume the proceeds, or to hold the bond despite the lower utility. It is intuitive that the disutility from waiting approaches 0 for bonds with very short maturities. Therefore, investors avoid paying the bid-ask spread for short-term bonds and never sell them prematurely. The endogenously determined maturity for which an investor experiencing a preference shock is indifferent between selling a bond and holding it until maturity, $\tau$, satisfies

\[
p(\tau) \times (1 - s) = e^{-(r + b) \times \tau}
\]

and is identical for both investor types.

Below, we only consider steady-state equilibria, in which neither prices nor aggregate wealth changes over time. An investor then has no incentive to change her (initially optimal) portfolio allocation without a preference shock. It is therefore sufficient to consider the investor’s decision problem at time $t = 0$, where each investor maximizes expected lifetime utility by choosing the amount of money invested into the money market account and into bonds with different maturities. We formally derive this decision problem and calculate first order conditions in Appendix A.1, compute equilibrium (ask) prices as a function of maturity in Appendix A.2, and show that markets clear, given these prices, in Appendix A.3.

\section*{B Clientele Effect}

In equilibrium, if the wealth of low-risk investors alone is not sufficient to buy all bonds, there arises a clientele effect related to the ones in Amihud and Mendelson (1986) and Beber et al. (2012): Low-risk investors buy only bonds with maturity above an endogenously determined $T_{\text{lim}}$, and high-risk investors buy only bonds with maturity below $T_{\text{lim}}$. Note that dealers (in aggregate) do not absorb any inventory. Hence, low-risk in-
vestors absorb the supply of long-term bonds and high-risk investors absorb the supply of short-term bonds. Proposition 1 summarizes the results on equilibrium prices and the clientele effect. For ease of exposition, we set \( r = 0 \).

Proposition 1. (Equilibrium prices and clientele effect)

Consider the case that \( \tau < T_{\text{lim}} \). The prices of illiquid bonds \( p(T) \) are given by

\[
p(T) = \begin{cases} 
\frac{b \times e^{-\lambda_H \times T} - \lambda_H \times e^{-b \times T}}{b - \lambda_H}, & \text{if } T \leq \tau \\
\epsilon^{-\lambda_H \times s \times (T - \tau)} \times p(\tau), & \text{if } \tau < T \leq T_{\text{lim}}, \\
\epsilon^{-\lambda_L \times \frac{1}{1 + \Delta_L(T_{\text{lim}})} \times (T - T_{\text{lim}})} \times p(T_{\text{lim}}), & \text{if } T_{\text{lim}} < T 
\end{cases}
\]

where \( \Delta_L(T_{\text{lim}}) \) denotes marginal utility of low-risk investors when investing in bonds with maturity \( T_{\text{lim}} \). \( \tau \) is the maturity for which investors are indifferent between selling the bond when experiencing a preference shock and holding it until maturity. In equilibrium, there arises a clientele effect that leads to low-risk investors investing only in long-term bonds with \( T > T_{\text{lim}} \) and high-risk investors investing in short-term bonds with \( T \leq T_{\text{lim}} \).

The endogenous maturity thresholds \( \tau \) and \( T_{\text{lim}} \), which are implicitly defined in equations (1) and (A-40) in Appendix A.3, and marginal utility of low-risk investors \( \Delta_L(T_{\text{lim}}) \) (see equation (A-4) in Appendix A.1) determine bond prices in equation (2). For given \( \tau \) and \( T_{\text{lim}} \), bond prices can be interpreted as follows: For very short-term bonds with maturities below \( \tau \), only high-risk investors act as buyers. They never sell the bond prior to maturity. Hence, the return must only compensate them for the expected utility loss through the preference shock (which occurs at a rate \( \lambda_H \)). Bonds with maturity between \( \tau \) and \( T_{\text{lim}} \) are also only purchased by high-risk investors. They optimally sell these bonds upon a preference shock before the maturity has decreased to \( \tau \). Therefore, the bonds’ instantaneous return must compensate them for the expected transaction costs \( \lambda_H \times s \). Only low-risk investors buy long-term bonds with maturities higher than \( T_{\text{lim}} \). They optimally sell the bonds upon a preference shock. Hence, they demand compensation for their expected transaction costs. In addition, long-term bonds have to compensate low-risk in-
vestors for their “outside option” of investing in shorter-maturity bonds: Bonds of maturity $T < T_{\text{lim}}$ generate a positive expected return for low-risk investors net of expected transaction costs, since they compensate for the (higher) expected transaction costs of high-risk investors. To induce low-risk investors to buy long-term bonds, these must offer a higher instantaneous return $\lambda_L \times \frac{s + \Delta L(T_{\text{lim}})}{1 + \Delta L(T_{\text{lim}})} > \lambda_L \times s$.

We prove Proposition 1 in Appendix A.2, where we also provide formulas for the (economically less interesting) case $T_{\text{lim}} \leq \tau$.

C Comparison to Amihud and Mendelson (1986) Type Models

Before proceeding with the predictions of our model for trading volume and liquidity premia, it is instructive to compare our model to that of Amihud and Mendelson (1986). With respect to the optimization problem, we endogenize the investor’s decision to sell assets as a reaction to the preference shock. With respect to assets, we consider a continuum of assets with different maturities, but identical bid-ask spreads. Amihud and Mendelson (1986) consider assets with identical (infinite) maturity but different bid-ask spreads. This gives rise to the clientele effect: investors select assets with high bid-ask spreads because they have low trading needs, and hence lower expected holding costs over a given holding period. In contrast, our clientele effect (also) applies for bonds with identical bid-ask spreads. The mechanism behind this generalization of the clientele effect is the investor’s endogenous decision to sell, which allows her to trade-off one type of illiquidity (the disutility of higher transaction costs) against another type of illiquidity (the disutility from awaiting the bond’s maturity). Hence, even if short-term bonds are not more “liquid” with respect to transaction costs, they are more “liquid” due to the lower disutility from awaiting their (closer) maturity. In contrast, assuming that investors are forced to sell assets immediately after a liquidity shock (as, e.g., in Amihud and Mendelson (1986), Ericsson and Renault (2006), and Kempf et al. (2012)) is a special case of our setting and corresponds to $b \to \infty$. In this case, the second source of illiquidity is irrelevant, and there
is no advantage from investing in short-term bonds.

III Hypotheses on Trading Volume and Liquidity Term Structure

A Trading Volume

We first present the model-implied relations between trading volume, maturity, and age. These relations are intuitive: First, bonds of short maturities are not sold prematurely, since the disutility from awaiting maturity is low. Second, the clientele effect (high-risk investors with strong trading needs only hold short-term bonds) translates into lower trading volumes for bonds with longer maturities. The first and second effects lead to a hump-shaped relation between maturity and trading volume. Third, an aged bond (formerly long-term but now short-term) is still partially locked up in the portfolios of investors with low trading needs. This leads to a lower trading volume of this bond compared to a young short-term bond. We are not aware of any other model that is both able to endogenously derive relations between maturity, age, and trading volume and predict term structures of liquidity premia.

The predictions regarding trading volume are summarized in the following proposition. We exclude trading volume from issuing activities in the primary market, which are exogenous in our setting, and focus on secondary-market trading volume. Since dealers do not hold inventory in aggregate, trading volume equals twice the volume sold by investors to dealers.\(^5\)

\(^5\)We look at turnover, i.e., trading volume in percent of the outstanding volume for each maturity, and not absolute trading volume to facilitate a comparison with the empirical literature (e.g., Hotchkiss and Jostova (2007)) and to provide a fair comparison between different maturities that differ in their outstanding amounts.
Proposition 2. (Trading volume)

Consider the case that $\tau < T_{\text{lim}}$.

1. Secondary-market turnover is hump-shaped in the time to maturity $T$, more specifically, it is 0 for $T < \tau$ and equals $2\lambda_L$ for $T > T_{\text{lim}}$. For $T$ with $\tau < T < T_{\text{lim}}$, turnover exceeds $2\lambda_L$.

2. For two bonds 1 and 2 that both have a remaining maturity $T$ with $\tau < T < T_{\text{lim}}$, but a different initial maturity $T_{\text{init},1} < T_{\text{init},2} > T_{\text{lim}}$, secondary-market turnover is higher for the younger bond 1 than for the older bond 2.

In the (less interesting) case that $T_{\text{lim}} \leq \tau$, high-risk investors never sell bonds prematurely, and turnover is determined by low-risk investors only. Hence, turnover is 0 for $T < \tau$, and equals $2\lambda_L$ for $T > \tau$. Then, no aging effect arises. We provide the proof of Proposition 2 (for general bid-ask spreads) in the Internet Appendix 1 (all Internet Appendices are available at www.jfqa.org).

We illustrate the relation between maturity and trading volume for a baseline parameter specification in Figure 1. Bid-ask spreads are 0.3% for all maturities $T$. High-risk investors experience preference shocks with a rate of $\lambda_H = 0.5$, i.e., on average 1 preference shock every 2 years. Low-risk investors experience half as many shocks ($\lambda_L = 0.25$).\textsuperscript{6} $b$ equals 2%, i.e., if a shock arises, investors’ time preference rate increases by 2%.

The dependence of trading volume on the distribution of bonds over the portfolios of low- and high-risk investors leads to the aging effect (second part of Proposition 2). Bonds with initial maturity $T_{\text{init}} < T_{\text{lim}}$ (dotted line in Figure 1) are only held by high-risk investors.

\textsuperscript{6}Our parameter values are comparable to Feldhütter (2012), who estimates for the U.S. corporate bond market that investors experience a preference shock once every 3 years.
investors. These investors sell the bonds when experiencing a preference shock if the remaining maturity $T$ is larger than $\tau$. Turnover thus equals $2\lambda_H$ for $T > \tau$ and drops to 0 for $T < \tau$.

The same intuition applies for low-risk investors and bonds with initial maturity $T_{\text{init}} > T_{\text{lim}}$ (dashed line in Figure 1) while their maturity $T$ is larger than $T_{\text{lim}}$. If these bonds reach a remaining maturity $T$ below $T_{\text{lim}}$, only high-risk investors purchase them. Hence, the bonds gradually move into the portfolios of high-risk investors, who suffer preference shocks at a higher rate. Therefore, turnover increases for decreasing maturity (until it drops to 0 at $\tau$). As a direct consequence, a bond with remaining maturity $T < T_{\text{lim}}$ has lower turnover if its initial maturity was larger than $T_{\text{lim}}$ (the bond is older), compared to a (younger) bond with remaining maturity $T$ and initial maturity $T_{\text{init}} < T_{\text{lim}}$.\footnote{Note that it is irrelevant by how much the initial maturity $T_{\text{init}}$ exceeds $T_{\text{lim}}$ since all bonds with $T_{\text{init}} > T_{\text{lim}}$ are initially bought by low-risk investors.}

The solid line in Figure 1 shows turnover for all bonds. It corresponds to the weighted average of the other two lines, with weights equal the proportion of bonds of remaining maturity $T$. Our model predictions are consistent with the aging effect discussed in Warga (1992) and empirically documented, e.g., in Fontaine and Garcia (2012) for U.S. Treasuries and Hotchkiss and Jostova (2007) for corporate bonds. Note, however, that our aging effect is cross-sectional, i.e., it compares two bonds with the same remaining maturity but different age. It therefore differs from the on-the-run/off-the-run effect, which describes the decreasing trading volume over a single bond’s life.\footnote{Vayanos and Wang (2007) provide an explanation for this effect based on coordination. In their model, it is more attractive for speculators to trade bonds that are expected to be more actively traded in the future. For that reason, liquidity concentrates in newly issued on-the-run bonds.} In the empirical analysis, we isolate the impact of aging not due to the pure on-the-run/off-the-run effect.
B The Term Structure of Liquidity Premia

To demonstrate the effect of illiquidity on the term structure of interest rates, we separately compute liquidity premia from ask prices \( p^{\text{ask}}(T) = p(T) \) and from bid prices \( p^{\text{bid}}(T) = (1 - s) \times p(T) \). Note that we focus on (traded) ask and bid prices in contrast to (artificially calculated) mid prices. However, to provide a link to the empirical literature, which primarily studies mid prices, we also present predictions for mid premia in our two main Figures 2 and 3. Liquidity premia are defined as the bond yield minus the risk free rate \( r \), i.e.,

\[
\text{illiq}^{\text{ask}}(T) = -\frac{\ln \left( p^{\text{ask}}(T) \right)}{T} - r = -\frac{\ln (p(T))}{T} - r, \\
\text{illiq}^{\text{bid}}(T) = -\frac{\ln \left( p^{\text{bid}}(T) \right)}{T} - r = -\frac{\ln (1 - s)}{T} - \frac{\ln (p(T))}{T} - r.
\]

The formulas for liquidity premia can be interpreted as distributing the "liquidity discount" over the time to maturity \( T \). Ask liquidity premia decrease to 0 for decreasing maturity: both the probability of a preference shock and the utility loss in case of a shock go to 0 for \( T \to 0 \). Hence, required returns go to 0.

Bid liquidity premia correspond to ask liquidity premia increased by bid-ask spreads \( s \). Distributing \( s \) over a shorter maturity yields the first summand for \( \text{illiq}^{\text{bid}} \), which goes to infinity for decreasing maturity \( T \) (as \( s \approx -\ln (1 - s) \) for small \( s \)). For increasing maturities, \( \text{illiq}^{\text{bid}} \) mimics the behavior of \( \text{illiq}^{\text{ask}} \), since \( s \) is distributed over increasing \( T \).

We summarize our model predictions regarding the term structure of liquidity premia in Proposition 3, which we prove (for general bid-ask spreads) in the Internet Appendix 1.

Proposition 3. (Term structure of liquidity premia)

1. The term structure of liquidity premia from ask prices \( \text{illiq}^{\text{ask}}(T) \) is monotonically increasing in time to maturity \( T \) for all \( T \) and goes to 0 for \( T \to 0 \). The term structure
flattens at $T_{\text{lim}}$, i.e.,

$$
\lim_{T \uparrow T_{\text{lim}}} \frac{\partial \text{illiq}^{\text{ask}}(T)}{\partial T} > \lim_{T \downarrow T_{\text{lim}}} \frac{\partial \text{illiq}^{\text{ask}}(T)}{\partial T}.
$$

2. The term structure of liquidity premia from bid prices $\text{illiq}^{\text{bid}}(T)$ is decreasing in $T$ at the short end.

3. Illiquidity spills over from short-term to long-term maturities: ceteris paribus, higher (lower) liquidity premia for $T \leq T_{\text{lim}}$ due to a higher (lower) liquidity demand of high-risk investors $\lambda_{H}$ lead to higher (lower) liquidity premia for maturities $T > T_{\text{lim}}$. The reverse effect does not hold, i.e., liquidity premia below the minimum of the old and the new $T_{\text{lim}}$ are unaffected by a change in $\lambda_{L}$.

The predictions in Proposition 3 are illustrated in Figure 2.

Insert Figure 2 about here.

Ask premia $\text{illiq}^{\text{ask}}(T)$ (solid lines) go to 0 for $T \rightarrow 0$ as the disutility from awaiting the bond’s maturity vanishes. The ask term structure $\text{illiq}^{\text{ask}}(T)$ flattens out quickly with a kink at $T_{\text{lim}}$ (which is hard to detect as the slope is already small for $T \uparrow T_{\text{lim}}$). The kink arises because low-risk and high-risk investors require different returns. High-risk investors require compensation for their (relatively high) expected transaction costs for bonds with maturity below $T_{\text{lim}}$. Low-risk investors require compensation for their (lower) expected transaction costs, plus compensation for investing in long-term bonds. Without the kink, high-risk investors would also invest in long-term bonds. This decrease of the slope corresponds to the convexity of the liquidity premium in Amihud and Mendelson (1986). It is also consistent with the empirical results of Huang, Sun, Tao, and Yu (2014), who find that investors with low liquidity needs on average hold more illiquid bonds with higher liquidity premia (our clientele effect), but demand less compensation than investors with higher trading needs would. Bid-premia $\text{illiq}^{\text{bid}}(T)$ (dashed lines) exhibit an inverse shape and, like ask premia, flatten out for increasing maturities.
We also illustrate the spillover effect from short-term to long-term liquidity premia in Figure 2. Thin lines depict the case where high-risk investors’ liquidity demand $\lambda_H$ is twice as large as in the baseline specification (thick lines). All other parameters remain unchanged. Although only high-risk investors are affected by this change, liquidity premia of all maturities increase. The economic rationale for this liquidity spillover is that low-risk investors would prefer short-term bonds over long-term bonds if long-term ask liquidity premia were lower than short-term premia. In terms of observables, the spillover corresponds to a (causal) impact of shocks in short-term liquidity premia to long-term liquidity premia. Therefore, our model provides a formal mechanism for liquidity transmission between different maturity segments, which Goyenko et al. (2011) empirically document for bid-ask spreads of U.S. Treasury bonds.

Figure 2 allows us to make an additional observation: If we average ask and bid prices to compute mid liquidity premia $illiq^{mid}(T)$, we get an inverse shape with large premia for very short maturities. Figure 2 depicts these mid premia in the dotted lines. Taking into account that empirically observed bond yield spreads are typically computed from such mid prices and incorporate a liquidity component, our model can shed light on the credit spread puzzle: empirically observed bond yield spreads are too high, especially at the short end, compared to what structural models such as Merton (1974) can explain (see, e.g., Huang and Huang (2012)).

IV Increasing Bid-Ask Spreads

Up until now, we assumed that bid-ask spreads are the same across all maturities. Empirical bid-ask spreads, however, display a systematic dependence on maturity: Bonds with longer time to maturity exhibit higher bid-ask spreads than bonds with shorter maturity. We therefore extend our model to accommodate bid-ask spreads $s(T)$ as a nonnegative, monotonically increasing function of maturity $T$. Doing so yields the same ask prices
as in Proposition 1 when we substitute the average bid-ask spread (i.e., $\int_{T}^{\tau} s(x) dx$ for $T < \tau < T_{\text{lim}}$ and $\int_{T_{\text{lim}}}^{\tau} s(x) dx$ for $T_{\text{lim}} < T$) for the constant bid-ask spread $s$. The propositions regarding trading volume and the term structure of liquidity premia are unaffected by our allowing for more general bid-ask spreads. The main effect of increasing bid-ask spreads is on the term structure of liquidity premia at the long end. To illustrate this behavior, we calibrate a parametric form of bid-ask spreads to observed bid-ask spreads (for details, see the Internet Appendix 4), and display the resulting liquidity term structure in Figure 3.

Figure 3 shows that increasing bid-ask spreads affect our hypotheses on the behavior of the liquidity term structure in two ways. First, ask premia illiq\text{ask}(T)$ (solid lines) now increase more strongly at the long end. This is due to the higher expected trading costs for longer maturities. Hence, the kink at $T_{\text{lim}}$ becomes more apparent.

Second, bid and mid premia (dashed and dotted lines) now exhibit an inverse shape only for short-term premia and increase at the long end. The increasing bid-ask spread curve therefore results in increasing term structures of liquidity for ask and U-shaped ones for bid and mid liquidity premia. Our model thus helps to reconcile conflicting evidence on the term structure of liquidity in the literature: Longstaff (2004) and Ericsson and Renault (2006) find a U-shaped or decreasing term structure for mid quotes, bond prices from Datastream (presumably also mid quotes), and average transaction prices obtained from insurance companies (National Association of Insurance Commissioners data). Amihud and Mendelson (1991) use ask prices net of brokerage fees, effectively tilting ask prices in the direction of mid quotes, and also find a decreasing term structure. In contrast, our model predicts an increasing term structure for ask quotes, which is the shape Dick-Nielsen et al. (2012) find for average quarter-end trade prices from TRACE. Since these prices are dominated by buy transactions at the ask quotes, as the numbers of observations in Panel A of Table 1 document, their result is also consistent with our model prediction. When ap-
plying a possibly time-varying mixture of bid and ask quotes, hump-shaped (Koziol and Sauerbier (2007)) or variable term structures (Kempf et al. (2012)) can arise. Hence, the empirically documented differences on the shape of the liquidity term structure can be explained in our model by whether most transactions occur at the bid or ask.

Last, the extension of the model to maturity-dependent bid-ask spreads $s(T)$ allows us to analyze spillover effects due to changing short-term bid-ask spreads. As illustrated in Figure 3 by the thin lines, an increase of short-term bid-ask spreads $s(T)$ for only $T \leq 1.25$ leads to an increase of liquidity premia of all maturities. Hence, an increase in short-term liquidity premia via either increased liquidity demand $\lambda_H$ (see Figure 2) or higher short-term bid-ask spreads can lead to higher long-term liquidity premia.

V Empirical Analysis

We now empirically test the predictions of our model, which we can summarize as follows: First, turnover is hump shaped. Second, for bonds with identical maturity but a different age, the older bond has a lower turnover compared to the younger bond. Third, ask liquidity premia are monotonically increasing in maturity at the short end and flatten out for longer maturities. Fourth, bid liquidity premia are monotonically decreasing at the short end and, depending on the shape of $s(T)$, flatten out or start increasing for longer maturities. Fifth, liquidity shocks spill over from the short end to the long end of the liquidity term structure. In the following, we first describe our data in Section A and formulate the empirical hypotheses in Section B. Sections C-E then contain the main empirical tests. For details on the data selection, we refer to Appendix B. We provide further details on the calculation of liquidity premia, the shape of the empirical bid-ask spread curve, and robustness checks in the Internet Appendices 3-5.
A Data

We use corporate bond transaction data from Enhanced TRACE. Obviously, corporate bond prices are driven by credit risk, which we do not capture in our model. Hence, one could argue that our model should rather be tested on the Treasury or Treasury Inflation Protected Security (TIPS) market. Nevertheless, we choose the corporate bond market for two reasons: First and most importantly, data quality, availability, and level of detail in Enhanced TRACE is superior by far to what is available for other bond types. For example, using Enhanced TRACE allows us to use price and volume information on time-stamped transactions separately for buyer-dealer trades and seller-dealer trades. We can therefore distinguish cleanly between transactions that occur at the ask price and those that occur at the bid price, which is central for our model. Also, the data cover a long time period, which allows us to test the liquidity spillover from the short end to the long end. Second, illiquidity plays a more important role for corporate bonds than for Treasury or agency bonds, and cross-sectional liquidity differences are especially pronounced. This should allow us to cleanly disentangle liquidity-related effects from other effects on the term structure of yield spreads.

Naturally, the validity of our results depends on appropriately capturing credit risk. We pay particular attention to isolating the liquidity effect in corporate bond prices via two completely independent, well-established approaches. Repeating the empirical analysis for individual rating classes further confirms that our results are not due to cross-sectional differences in credit risk.

We use transaction data from TRACE, information on rating, maturity, coupon, outstanding notional, and other features from Reuters and Bloomberg, and focus on plain-vanilla investment grade bonds. The data selection procedure is described in Appendix B. We use Treasury yields as the risk-free interest-rate curve and employ swap rates instead as a robustness check in the Internet Appendix 5.
To isolate the liquidity component in bond yields, we apply two methodologies. In the first approach (difference approach), we compute the liquidity premium as the difference between the observed bond yield and the yield of a theoretical bond with identical promised cash flows, but which is only subject to credit risk. We compute the price of this theoretical bond using risk-free rates and a term structure of CDS premia. This approach is in line with, e.g., Longstaff (2005), and has the advantage that the resulting liquidity premium \( \text{illiq}_{\text{diff}}^{\text{ask/bid}}(T) \) does not depend on a specific proxy for bond illiquidity.

Since several papers have shown that CDS premia are not completely free from liquidity concerns (see, e.g., Arakelyan and Serrano (2016), Biswas, Nikolova, and Stahel (2015)), we implement an alternative approach that precludes an influence from liquidity term structure effects in CDS curves on our results. In this second approach (regression approach), we follow Dick-Nielsen et al. (2012) and identify the liquidity component in bond yields by regressing bond yield spreads on a specific liquidity proxy: the average of the Amihud (2002) liquidity measure, imputed roundtrip costs as in Feldhüttter (2012), and their intra-month standard deviations. Within the regression, we differentiate between bid and ask yields and monthly duration buckets \( T_m \in \{\frac{1}{12}, \frac{2}{12}, ..., \} \) with \( \{T_m \leq T < T_m + \frac{1}{12}\} \).9

The resulting liquidity premium \( \text{illiq}_{\text{reg}}^{\text{ask/bid}}(T_m) \) is then computed using the coefficient estimates from the regression and the monthly average liquidity proxy value. We describe the difference and the regression approaches in more detail in the Internet Appendix 3.

B Hypotheses

Our model predicts a nonlinear relation between maturity \( T \) and bid and ask liquidity premia. More formally, it predicts that the sensitivity of liquidity premia to maturity \( T \) is different for short- and long-term bonds. To test these relations, we employ piece-

\[9\text{Since our theoretical predictions are for zero coupon bonds, but traded bonds are mainly coupon bonds, we use duration instead of time to maturity in our empirical tests. We obtain similar results when using the time to maturity as an explanatory variable.}\]
wise linear regressions that explicitly allow for such a different sensitivity for maturities below and above a breakpoint $\theta$:

\[
\text{illiq}^{\text{ask}}(T) = \alpha^{\text{ask}} + \beta^{\text{ask}1}_{1} \mathbb{1}_{\{T \leq \theta\}} \times (T - \theta) + \beta^{\text{ask}2}_{2} \mathbb{1}_{\{T > \theta\}} \times (T - \theta) + \gamma^{\text{ask}} \text{CONTROLS} + \varepsilon,
\]

\[
\text{illiq}^{\text{bid}}(T) = \alpha^{\text{bid}} + \beta^{\text{bid}1}_{1} \mathbb{1}_{\{T \leq \theta\}} \times (T - \theta) + \beta^{\text{bid}2}_{2} \mathbb{1}_{\{T > \theta\}} \times (T - \theta) + \gamma^{\text{bid}} \text{CONTROLS} + \varepsilon,
\]

where $\text{illiq}^{\text{ask}}(T)$ ($\text{illiq}^{\text{bid}}(T)$) is the liquidity premium computed from ask (bid) prices, $T$ is the duration of the bond, and $\varepsilon$ is an error term. When isolating liquidity premia with the difference approach ($\text{illiq}_{\text{diff}}$), we include the CONTROLS bond age, average numerical rating, and the logarithm of the outstanding amount.\(^{10}\) Since we have observations for each bond $j$ and each buy/sell trade at time $t$, equation (5) is estimated as a panel regression and we include firm and month fixed effects. For the analysis of liquidity premia isolated via the regression approach ($\text{illiq}_{\text{reg}}$), we account for a potential impact of the control variables already in the first-step regression (for details, see the Internet Appendix 3). For this approach, we have one average liquidity premium for each monthly duration bucket $T_m$ and, thus, regression (5) is purely cross-sectional. We employ a wide range of exogenous breakpoints $\theta$ between 3 months and 3 years and additionally compute an endogenous breakpoint $\theta^*$ by minimizing the sum of squared residuals.\(^{11}\)

If our hypotheses regarding the liquidity term structure are confirmed, we expect the following behavior. We should find positive estimates for $\beta^{\text{ask}1}_1$ and $\beta^{\text{ask}2}_2$ as the slope of the ask liquidity premium term structure is positive for all maturities. Because our model

\(^{10}\)Edwards et al. (2007) report a dependence of transaction costs on age and outstanding volume which is not directly captured by our model.

\(^{11}\)Naturally, our model does not imply that our predictions hold for every possible exogenous duration breakpoint. Also, different estimated breakpoints on the bid and the ask side are not in contradiction to our model.
predicts a flattening term structure, we expect $\beta_1^{\text{ask}}$ to be larger than $\beta_2^{\text{ask}}$. For bid premia, we should find negative estimates for $\beta_1^{\text{bid}}$. As Figure 3 shows for the empirically calibrated bid-ask spread curve, bid premia are relatively flat but slightly increasing at the long end. Therefore, we expect $\beta_2^{\text{bid}}$ to be either not significantly different from 0 or slightly positive.

A similar intuition holds for trading volume. There, we use a panel regression of the form

$$ \text{turnover}(T_{j,t}) = \alpha + \beta_1 1_{\{T_{j,t} \leq \theta\}} \times (T_{j,t} - \theta) + \beta_2 1_{\{T_{j,t} > \theta\}} \times (T_{j,t} - \theta) + \beta_3 \text{AGE}_{j,t} + \gamma \text{CONTROLS}_{j,t} + \varepsilon_{j,t}, $$

where turnover$(T_{j,t})$ is the turnover of bond $j$ at time $t$ with duration $T_{j,t}$, $\theta$ is the breakpoint, $\text{AGE}_{j,t}$ is bond age in years, and $\text{CONTROLS}_{j,t}$ again include the average numerical rating, the logarithm of the outstanding amount, and fixed effects. We expect positive estimates for $\beta_1$, negative estimates of $\beta_2$, and negative estimate for $\beta_3$, since our model predicts a lower trading volume for an older but otherwise equal bond compared to a younger one.

Last, our model predicts that liquidity shocks spill over from the short end to the long end of the liquidity premia term structure, but not vice versa. We use a vector autoregression (VAR) analysis with a lag length of 2 (which is sufficient via the AIC and BIC):

$$ \begin{align*}
\text{illiq}_{t}^{\text{ask}}(T < \theta) &= \alpha_{\text{short}}^{\text{ask}} + \sum_{i=1}^{2} \phi_{i,\text{short}}^{\text{ask}} \text{illiq}_{t-i}^{\text{ask}}(T < \theta) + \sum_{i=1}^{2} \beta_{i,\text{long}}^{\text{ask}} \text{illiq}_{t-i}^{\text{ask}}(T \geq \theta) + \varepsilon_t, \\
\text{illiq}_{t}^{\text{bid}}(T < \theta) &= \alpha_{\text{short}}^{\text{bid}} + \sum_{i=1}^{2} \phi_{i,\text{short}}^{\text{bid}} \text{illiq}_{t-i}^{\text{bid}}(T < \theta) + \sum_{i=1}^{2} \beta_{i,\text{long}}^{\text{bid}} \text{illiq}_{t-i}^{\text{bid}}(T \geq \theta) + \varepsilon_t, \\
\text{illiq}_{t}^{\text{ask}}(T \geq \theta) &= \alpha_{\text{long}}^{\text{ask}} + \sum_{i=1}^{2} \beta_{i,\text{short}}^{\text{ask}} \text{illiq}_{t-i}^{\text{ask}}(T < \theta) + \sum_{i=1}^{2} \phi_{i,\text{long}}^{\text{ask}} \text{illiq}_{t-i}^{\text{ask}}(T \geq \theta) + \varepsilon_t, \\
\text{illiq}_{t}^{\text{bid}}(T \geq \theta) &= \alpha_{\text{long}}^{\text{bid}} + \sum_{i=1}^{2} \beta_{i,\text{short}}^{\text{bid}} \text{illiq}_{t-i}^{\text{bid}}(T < \theta) + \sum_{i=1}^{2} \phi_{i,\text{long}}^{\text{bid}} \text{illiq}_{t-i}^{\text{bid}}(T \geq \theta) + \varepsilon_t,
\end{align*} $$
where $\text{illiq}^{\text{ask/bid}}_t(T < \theta)$ is the average liquidity premium computed from ask/bid prices across all short-term bonds (with a duration below breakpoint $\theta$) in month $t$, $\text{illiq}^{\text{ask/bid}}_t(T \geq \theta)$ is the average liquidity premium computed from ask/bid prices across all long-term bonds (with a duration of $\theta$ or above), $\phi_{i,\text{short}}^{\text{ask/bid}}(\phi_{i,\text{long}}^{\text{ask/bid}})$ measures the autocorrelation of the short-term (long-term) liquidity premium of order $i$, and $\beta_{i,\text{long}}^{\text{ask/bid}}(\beta_{i,\text{short}}^{\text{ask/bid}})$ measures the spillover of liquidity shocks from long-term bonds to short-term bonds (short-term bonds to long-term bonds) with lag $i$. If our hypotheses regarding the liquidity spillovers across the term structure hold, we should find insignificant estimates of $\beta_{i,\text{long}}^{\text{ask/bid}}$, but positive estimates of $\beta_{i,\text{short}}^{\text{ask/bid}}$.

### C Term Structure of Liquidity Premia

We first illustrate the average term structures of ask and bid liquidity premia together with the corresponding term structures predicted by our model in Figure 4. Visual inspection of Graphs A and B suggests that our main hypotheses regarding liquidity premia hold for both approaches to measure liquidity premia. Ask liquidity premia are mostly increasing in maturity, while bid liquidity premia exhibit an inverse shape. At the long end, both bid and ask premia slightly increase with maturity.

Insert Figure 4 about here.

We now formally explore the effect of maturity on bond liquidity premia and estimate equation (5) for five different exogenous specifications of the breakpoint $\theta$ between 3 months and 3 years, and for the endogenously determined breakpoint $\theta^*$. Standard errors are clustered by firm as suggested by Petersen (2009). The results of the panel regression are given in Table 1.

Insert Table 1 about here.
Panels A and B of Table 1 confirm our hypotheses regarding liquidity premia. Irre-
spective of the way we measure liquidity premia, we find that the estimates for the slope
at the short end, \( \beta_1^{\text{ask}} \), are always positive and significant for ask liquidity premia for 9 out
of 10 specifications of the exogenous breakpoint \( \theta \). The estimates for the slope at the long
end, \( \beta_2^{\text{ask}} \), are always positive and again significant for 9 out of the 10 specifications. They
are consistently smaller than the estimates for \( \beta_1^{\text{ask}} \) by more than a factor of ten. This re-
lation indicates a much higher slope at the short end. When we formally test this relation,
we obtain an always negative difference between the long and the short end which is signif-
icant in 9 out of the 10 cases. When estimating the breakpoint \( \theta^* \) endogenously, the slope
is in both cases significantly positive at the short end and (significantly) flatter for longer
maturities \( T > \theta^* \).

For bid liquidity premia, we obtain negative and significant estimates for the slope
at the short end for 8 out of the 10 specifications of the exogenous breakpoint. Consistent
with the shape of the predicted bid curve in Figure 4, 9 out of the 10 estimates for the
slope at the long end are positive but none of them is significant. This implies a relatively
flat term structure at the long end. When we again test formally for differences between
the long and the short end, the differences are positive in 9 and significant in 8 out of the
10 cases. The results for the endogenously estimated breakpoints \( \theta^* \) are the same: a signific-
antsly negative slope at the short end and a relatively flat term structure for maturities
above the breakpoint \( \theta^* \).

For \( \text{illiq}^{\text{ask/bid}}_{\text{diff}} \), the impact of the control variables age, rating and outstanding amount
is also as expected whenever significant.\(^{12}\)

\(^{12}\)As a robustness check, we follow Bessembinder, Kahle, Maxwell, and Xu (2009) and only consider
institutional-sized bond trades with transaction volumes at or above $100,000. The results are similar
to those in Panel A of Table 1. We do not focus exclusively on institutional-sized trades in the standard
specification since larger trades in corporate bond markets are associated with lower transaction costs
(Edwards et al. (2007)). Therefore, including both retail-sized and institutional-sized trades should allow
us to disentangle the liquidity related component in bond yields from other effects more cleanly. More-
Overall, the results of the regression analysis confirm our model predictions. Ask liquidity premia are monotonically increasing with a decreasing slope, while bid liquidity premia are decreasing for short maturities and flatten out at the long end.

D Turnover Analysis

To formally explore the hypotheses regarding secondary-market trading volume, we consider two subsamples. First, we use all transactions available in TRACE, standardized with the outstanding amount of the bond under consideration. Second, we exclude bonds immediately around changes in their outstanding volume (through new issues, reopenings, and bond repurchases) since we do not consider these events in our model. When bonds are newly issued, they are often first held by dealers, who distribute them to clients and other dealers.\textsuperscript{13} Hence, the time interval around new issues of bonds might consist of multiple inter-dealer trades. We therefore exclude transactions 2 months prior to a new issue and 6 months following the issue, and denote this sample by Excl[-2,+6].\textsuperscript{14} Since newly issued bonds are excluded, this sample should also be less affected by the on-the-run/off-the-run effect.

We now apply our piecewise linear panel regression approach with age as an additional explanatory variable according to equation (6). Since turnover cannot be calculated on a trade-by-trade basis, we aggregate trading volume for each bond and calendar month over, Schestag, Schuster, and Uhrig-Homburg (2016) show that transaction costs in retail-sized trades are closely connected to transaction costs in institutional-sized trades and thus, small trades contain valuable information.\textsuperscript{13} As Table B1 shows, buying and selling volume do not fully add up. In a robustness check, we repeat the regression separately for the buy and the sell side to exclude the possibility that primary market allocations and the dealers’ portfolio imbalances create the hump-shape. The results are virtually identical on both sides.\textsuperscript{14} We exclude the time 2 months before changes in the amount outstanding mainly because of trades taking place in connection with bond repurchases that are typically announced about a month in advance.
and compute average daily turnover to account for a different number of business days per month. We winsorize turnover at the 1% and 99% quantile. The regression results are displayed in Table 2.

Table 2 confirms our model predictions regarding the hump-shaped turnover. For both subsamples, trading volume first increases strongly: the factor loadings for the slope at the short end, \( \beta_1 \), are positive and significant whenever we consider breakpoints below 2 years. Following the breakpoint, trading volume decreases slowly, and the difference between the slopes at the short end and at the long end is significant in 7 out of the 10 specifications of the exogenous breakpoint and for both endogenously estimated breakpoints. The negative and significant loading for age in all specifications is consistent with our prediction of a lower trading volume for older but otherwise equal bonds. The results for the outstanding amount are also as expected: bonds with a higher outstanding volume are more liquid, and thus display a higher trading volume.\(^{15}\)

**E Spillover Analysis**

Finally, we formally test the hypotheses regarding spillovers between the short and the long end of the liquidity term structure. To compute an aggregate time series of liquidity premia at the long and the short end of the term structure, we proceed as follows. First, we fix an exogenous breakpoint \( \theta \) between 3 months and 3 years. Second, we winsorize trade-specific liquidity premia \( \text{illiq}^{\text{ask/bid}} \text{diff}(T) \) at the 1% and 99% quantile in each observation month \( t \), and then take averages to compute the short-term liquidity premium in

\(^{15}\)In an unreported analysis, we examine the dependence of trading volume on credit quality. We expect that selling incentives are stronger, and hence \( \tau \) is smaller and turnover at the very short end is higher for riskier bonds. Employing double-sorted portfolios based on liquidity and rating, we find (weak) evidence for this conjecture.
month \( t \) for bonds with duration \( T < \theta \). We proceed in the same way to compute the long-term liquidity premium.\(^{16}\) Following Goyenko et al. (2011), we de-trend both time series by removing a time trend and the square of the time trend. We then run the time series regression in equation (7) and display the results in Table 3.

Table 3 shows that the data confirm our predictions regarding the unilateral liquidity spillover from short-term to long-term bonds. In Panel A, the short-term liquidity premium is the dependent variable, and the explanatory variables are the lagged short-term and long-term liquidity premia. We find that no single estimate of \( \beta_{1,\text{long}} \) is significantly different from 0, and that we can neither reject the joint hypotheses that both coefficients \( \beta_{1,\text{long}} \) and \( \beta_{2,\text{long}} \) are equal to 0, nor that their sum is equal to 0. In Panel B, the long-term liquidity premium is the dependent variable, and the lagged short-term and long-term liquidity premia are the explanatory variables. There, we find that 7 out of 10 coefficient estimates of the first lag \( \beta_{1,\text{short}} \) are individually significant (for the bid and the ask side). For the second lag, 1 additional parameter is significantly positive. Moreover, we can reject the joint hypothesis that both coefficient estimates \( \beta_{1,\text{short}} \) and \( \beta_{2,\text{short}} \) are equal to 0, or that they sum up to 0 in 8 out of 10 cases.

Taken together, these findings are clear evidence for a spillover of liquidity shocks from the short end to the long end of the term structure, but not in the reverse direction.

\(^{16}\)To ensure that the duration reflected by short-term and long-term liquidity premia is stable, we first calculate a trade-size weighted average for bonds within the same monthly duration bucket. We then calculate the (unweighted) average across all duration buckets that belong to the short- and long-term segments, respectively.
VI Summary and Conclusion

In this paper, we develop an equilibrium model that captures the relation between maturity and liquidity for finite maturity assets. Our model assumptions are parsimonious: We only consider two highly realistic frictions prevalent in bond markets, positive bid-ask spreads (charged by market makers) for bonds of all maturities, and investors with different probabilities of experiencing a liquidity shock. Based on these minimal assumptions, we show that a clientele effect arises: investors more likely to experience liquidity shocks only invest in short-term bonds, investors less likely to experience liquidity shocks only invest in long-term bonds. Since we depart from the previous literature by endogenizing the investors’ reaction to a liquidity shock, we can also show that investors only sell intermediate- to long-term bonds. In terms of observables, our model predicts a hump-shaped term structure of trading volume, opposing shapes for the term structure of liquidity premia in bid and ask prices, and a spillover of liquidity shocks from short-term to long-term bonds (but not vice versa). We verify our model predictions in an extensive empirical analysis of corporate bonds with a wide range of maturities, thus reconciling conflicting empirical evidence on the term structure of liquidity premia.

Our results have important implications for investors, policy makers, and corporations alike. From the perspective of the issuing corporation, our results reveal that by choosing a certain maturity, issuers also choose a certain investor base: high-risk investors when issuing short-term bonds, low-risk investors when issuing long-term bonds. Naturally, the investor base affects an issuer’s cost of capital. Beyond this maturity-specific effect, the liquidity spillover implies that even the cost of capital of corporations who issue long-term debt is negatively affected by deteriorating short-term liquidity. This impact is further amplified through the interplay of liquidity and credit risk. He and Xiong (2012) show that higher corporate bond liquidity premia decrease the issuer’s optimal default boundary. Therefore, any shocks that lead to higher short-term liquidity premia increase
credit risk both at the individual firm level and across all firms (and thus financial stability), irrespective of the maturity of their debt. Hence, from a policy perspective, it is important to recognize that the effectiveness of policy interventions crucially depends on the dedicated maturity segment. Measures targeting short-term investors such as liquidity buffers for mutual funds under the Investment Company Act or liquidity risk tools for banks under Basel III decrease the cost of capital, and increase financial stability, for the entire economy. In contrast, measures targeting long-term investors such as, e.g., liquidity buffers for insurance companies under Solvency II, only affect the long-term segment.

Last, our results highlight the importance of the term structure of bid-ask spreads for the functioning of (secondary) bond markets. For instance, an artificial increase of transaction costs, e.g., through a fixed financial transaction tax, shifts the maturity limit below which investors never sell bonds to higher values. Hence, our model predicts that the market for short-term securities dries up when bid-ask spreads increase. Conversely, any measures that improve short-term liquidity positively affect liquidity across all maturities. Policy interventions that stabilize security demand (e.g., the European Central Bank’s Corporate Sector Purchase Program) or subsidized dealer systems that directly decrease transaction costs should therefore predominantly address short-term securities.
References


Appendix A: Equilibrium Prices and Theoretical Derivations

We first derive the investor’s individual optimization problem in Appendix A.1. To avoid deriving all results for constant and monotonically increasing bid-ask spreads separately, we directly focus on the general case of a monotonically increasing bid-ask spread function \( s(T) \). Our model can be viewed as a continuous modification of a linear exchange model (see Gale (1960)) for which unique solutions exist. The equilibrium mechanism is similar to the ones in Amihud and Mendelson (1986) and Beber et al. (2012). For a given model parameter set \((\lambda_L, \lambda_H, w_L, w_H, b, T_{\text{max}})\) and bid-ask spread function \( s(T) \), equilibrium prices \( p(T) \), which we calculate in Appendix A.2, depend on critical maturities \( \tau \) (below which it is not optimal to sell bonds after a preference shock) and \( T_{\text{lim}} \) (below which it is optimal for low-risk investors not to invest, and above which it is optimal for high-risk investors not to invest). Simultaneously, critical maturities depend on equilibrium prices. We use a market clearing argument in Appendix A.3 to calculate \( T_{\text{lim}} \) and iterate until convergence over the calculation of equilibrium prices \( p(T) \), \( \tau \), and \( T_{\text{lim}} \). We finally prove Propositions 2 and 3 in the Internet Appendix 1 and demonstrate in the Internet Appendix 2 that the assumptions used in formulating the investor’s optimization problem hold.

Appendix A.1 – Optimization Problem

Conditional on a liquidity shock at time \( \tilde{T}_i \), total utility from consumption for an (infinitesimally small) investor of group \( i \) is given by
\[
  u_i(c) = \int_0^{	ilde{T}_i} e^{-r \times t} c_t \, dt + \int_{\tilde{T}_i}^{\infty} e^{-r \times t} c_t - (r + b) \times (t - \tilde{T}_i) c_t \, dt.
\]
Since investors are risk-neutral, their expected utility function has an additive structure and they want to invest either nothing or the maximum amount possible in a particular maturity (see, e.g., Feldhütter (2012)). The investor’s decision to invest in a particular
maturity hence neither depends on her wealth nor on her holdings in other maturities. Moreover, an investor who initially invests in a bond of some maturity $T$ also re-invests in a bond with this maturity if her old bond matures since her optimization problem is unaffected by a possible change in her wealth. In summary, each investor chooses an initially optimal allocation strategy when she first enters the market and has no incentive to change her portfolio prior to a preference shock.

Going forward, we make two assumptions in deriving the investor’s optimization problem. First, we assume that in the case of a preference shock, it is optimal to either sell the bond immediately or hold it until maturity. Second, we assume that it is never optimal to sell bonds when no preference shock has occurred. In the Internet Appendix 2, we derive general conditions under which these assumptions hold. Particularly, they always hold for constant bid-ask spreads $s$.

In steady state, the aggregate wealth of all type-$i$ investors has to be constant. Therefore, capital gains of any investor group $i$ cannot exceed the rate at which members of the respective investor group leave the market (otherwise, the wealth of investor group $i$ would grow to infinity). In equilibrium, $r + b$ is an absolute upper bound for the growth rate of wealth such that we assume $r + b < \lambda_L < \lambda_H$.

As neither aggregate wealth nor bond supply changes over time, prices of bonds for a given maturity are constant over time. We therefore only consider the decision problem at time $t = 0$, when each type-$i$ investor maximizes her expected utility $E[u_i(c)]$ from consumption by choosing the amount of money $X_i$ invested into the money market account ($X_i(0)$) and into bonds with maturity $T$ between 0 and $T_{max}$ ($X_i(T)$). Short sales are not allowed, so $X_i(T) \geq 0$, $\forall T \in [0, T_{max}]$. Hence, a type-$i$ investor solves the following opti-
mization problem:

\[
\text{(A-1)} \\
m_{x_i} = \max \left\{ \int_0^{T_{\text{max}}} X_i(T) \times \sum_{j=1}^{\infty} \frac{1}{p(T)} \times (1 - s(T \times j - \tilde{T}_i)) \times p(T \times j - \tilde{T}_i) \times e^{-r \times \tilde{T}_i} \times \mathbb{1}_{\{T \times (j-1) < \tilde{T}_i < T \times j - \min(\tau, T)\}} dT \right. \\
+ \int_0^{T_{\text{max}}} X_i(T) \times \sum_{j=1}^{\infty} \frac{1}{p(T)} \times e^{-r \times \tilde{T}_i - (r+b) \times (T \times j - \tilde{T}_i)} \times \mathbb{1}_{\{T \times j - \min(\tau, T) \leq \tilde{T}_i \leq T \times j\}} dT \\
+ X_i(0) \right\},
\]

where \(\tau\) satisfies a version of equation (1) adapted to the more general definition of bid-ask spreads \(s(T)\):

\[
\text{(A-2)} \quad p(\tau) \times (1 - s(\tau)) = e^{-(r+b) \times \tau}.
\]

The first summand in expression (A-1) denotes utility of consumption from bonds which the investor sells to the dealer at the bid price \((1 - s(T \times j - \tilde{T}_i)) \times p(T \times j - \tilde{T}_i)\) immediately after a preference shock. The amount invested in bonds \(X_i(T) \times \frac{1}{p(T)}\) grows for as many investment rounds \(j\) as the investor (re-)invests in the bond until the preference shock and thereby in each round collects the price difference between the notional value of the bond and the price of the bond \(p(T)\). The second summand gives the utility of consumption from bonds which the investor holds after the preference shock until their maturity date. The third summand measures the utility from cash invested in the money market account.

The investor’s budget constraint is \(W_i = \int_0^{T_{\text{max}}} X_i(T) dT + X_i(0)\). Simplifying expression (A-1), taking expectations, and replacing \(X_i(0)\) via the budget constraint yields the
following optimization problem:

\[
\text{(A-3)} \quad \max_{X_i} \left\{ \int_0^{T_{\max}} X_i(T) \times \frac{\lambda_i \times e^{\lambda_i \times T}}{p(T) \times e^{r \times T} \times e^{\lambda_i \times T} - 1} \times \int_{\min(\tau, T)}^T p(x) \times e^{r \times x} \times (1 - s(x)) \times e^{-\lambda_i \times (T-x)} \, dx \, dT \\
+ \int_0^{T_{\max}} X_i(T) \times \frac{\lambda_i \times (1 - e^{(\lambda_i - b) \times \min(\tau, T)})}{(1 - p(T) \times e^{r \times T} \times e^{\lambda_i \times T}) \times (\lambda_i - b)} \, dT \\
+W_i - \int_0^{T_{\max}} X_i(T) \, dT \right\}.
\]

Taking partial derivatives with respect to each \(X_i(T)\) yields the marginal utility of holding bonds with maturity \(T\) for a type-\(i\) investor:

\[
\text{(A-4)} \quad \frac{\partial E[u_i(c)]}{\partial X_i(T)} = \frac{\lambda_i \times e^{\lambda_i \times T}}{p(T) \times e^{r \times T} \times e^{\lambda_i \times T} - 1} \times \int_{\min(\tau, T)}^T p(x) \times e^{r \times x} \times (1 - s(x)) \times e^{-\lambda_i \times (T-x)} \, dx \\
+ \frac{\lambda_i \times (1 - e^{(\lambda_i - b) \times \min(\tau, T)})}{(1 - p(T) \times e^{r \times T} \times e^{\lambda_i \times T}) \times (\lambda_i - b)} - 1 =: \Delta_i(T).
\]

The fact that marginal utility does not depend on \(X_i\) simplifies the equilibrium: As investors are indifferent between all bonds they invest in, the marginal utility of these bonds must be equal. As marginal utility does not depend on \(X_i\), it is sufficient to consider whether an investor buys a bond at all. Given that the investor buys the bond, she is indifferent on how she distributes her wealth across all bonds she invests in.

Equation (A-4) also shows that the time preference rate \(r\) which applies prior to the liquidity shock does not affect the investor’s optimization problem. To see why, we rewrite bond prices as \(p(T) = e^{-r \times T} \times q(T)\). Here, \(q(T)\) is the discount of an illiquid bond compared to the price of a perfectly liquid bond \(e^{-r \times T}\). Substituting \(q(T) = e^{r \times T} \times p(T)\) into equations (A-2) and (A-4) would lead to an identical optimization problem independent of \(r\). To simplify notation, we therefore set \(r = 0\) in the following analysis.
Appendix A.2 – Equilibrium Prices and Clientele Effect

Marginal utility for holding bonds is larger for low-risk investors than for high-risk investors. Marginal utility of the money market account is equal for both. Hence, the allocation that high-risk investors buy bonds and, at the same time, low-risk investors invest in the money market account cannot be an equilibrium. We therefore focus on the most general remaining allocation: both high- and low-risk investors hold bonds, and high-risk investors additionally invest in the money market account.

Below, we show that given the calculated equilibrium prices, there exists a limiting maturity $T_{\text{lim}}$ such that low-risk investors buy only bonds with maturity between $0$ and $T_{\text{lim}}$ (clientele effect). The equilibrium conditions are then given by

$$\Delta_H(T) = 0 \quad \text{for all } T \in (0, T_{\text{lim}}], \tag{A-5}$$

$$\Delta_L(T) = \Delta_L(T_{\text{lim}}) \quad \text{for all } T \in (T_{\text{lim}}, T_{\text{max}}], \tag{A-6}$$

where $\Delta_i(T)$ is defined as in equation (A-4). High-risk investors are indifferent between holding bonds with a maturity up until $T_{\text{lim}}$ and the money market account, low-risk investors are indifferent between buying bonds with maturities between $T_{\text{lim}}$ and $T_{\text{max}}$.

Calculation of Equilibrium Prices

For given limiting maturities $\tau$ and $T_{\text{lim}}$, the conditions in equations (A-5) and (A-6) lead to analytical formulas for $p(T)$. In the following, items (i)-(iii) consider the case $\tau < T_{\text{lim}}$, which we deal with in Proposition 1. Items (i), (iv), and (v) are for the economically less interesting case that $T_{\text{lim}} \leq \tau$.

(i) For $T \leq \min(\tau, T_{\text{lim}})$, the integral term of equation (A-4) is 0. Using the first
order condition (A-5), we get

\[
\Delta_H(T) = \frac{\lambda_H \times (1 - e^{(\lambda_H - b)\times T})}{(1 - p(T) \times e^{\lambda_H \times T}) \times (\lambda_H - b)} - 1 \overset{!}{=} 0. \tag{A-7}
\]

Solving condition (A-7) for \(p(T)\) directly yields

\[
p(T) = \frac{b \times e^{-\lambda_H \times T} - \lambda_H \times e^{-b \times T}}{b - \lambda_H} \quad \text{for } T \leq \min(\tau, T_{\text{lim}}). \tag{A-8}
\]

(ii) For \(\tau < T \leq T_{\text{lim}}\), again using equation (A-4), the first order condition (A-5) evaluates to

\[
\Delta_H(T) = \frac{\lambda_H \times e^{\lambda_H \times T}}{p(T) \times e^{\lambda_H \times T} - 1} \times \int_{\tau}^{T} p(x) \times (1 - s(x)) \times e^{-\lambda_H \times (T-x)} \, dx \\
+ \frac{\lambda_H \times (1 - e^{(\lambda_H - b)\times \tau})}{(1 - p(T) \times e^{\lambda_H \times T}) \times (\lambda_H - b)} - 1 \overset{!}{=} 0. \tag{A-9}
\]

The solution of this integral equation is given as

\[
p(T) = e^{-\lambda_H \int_{\tau}^{T} s(x) \, dx} \times p(\tau) \quad \text{for } \tau < T \leq T_{\text{lim}}, \tag{A-10}
\]

which can be verified by plugging in (A-10) into (A-9). It is instructive to note that (A-10) corresponds to the market value of a defaultable bond with a default intensity \(\lambda_H\) and a “recovery-rate” of \((1 - s(T))\) when using the “recovery of market value assumption” in Duffie and Singleton (1999).

(iii) For \(\tau < T_{\text{lim}} < T\), we insert equation (A-4) into the first order condition for the low-risk investors (A-6) and get

\[
\Delta_L(T) = \frac{\lambda_L \times e^{\lambda_L \times T}}{p(T) \times e^{\lambda_L \times T} - 1} \times \int_{\tau}^{T} p(x) \times (1 - s(x)) \times e^{-\lambda_L \times (T-x)} \, dx \\
+ \frac{\lambda_L \times (1 - e^{(\lambda_L - b)\times \tau})}{(1 - p(T) \times e^{\lambda_L \times T}) \times (\lambda_L - b)} - 1 \overset{!}{=} \Delta_L(T_{\text{lim}}). \tag{A-11}
\]
By plugging in

\[ p(T) = e^{-\lambda_L \times \int_{T}^{T_{\text{lim}}} s(x) \, dx + \Delta_L(T_{\text{lim}}) \times (T - T_{\text{lim}}) \, \frac{1}{1 + \Delta_L(T_{\text{lim}})}} \times p(T_{\text{lim}}) \quad \text{for } \tau < T_{\text{lim}} < T, \]

we show that (A-12) solves the integral equation (A-11).

(iv) For \( T_{\text{lim}} < T \leq \tau \), we can ignore the first term of equation (A-4) and then again employ the first order condition for the low-risk investors (A-6) to get

\[ \Delta_L(T) = \frac{\lambda_L \times (1 - e^{\lambda_L \times T})}{(1 - p(T) \times e^{\lambda_L \times T}) \times (\lambda_L - b)} - 1 = \Delta_L(T_{\text{lim}}). \]

Rearranging terms directly yields

\[ p(T) = e^{-T \times \lambda_L \times \left(1 - \frac{\lambda_L \times (1 - e^{T \times (\lambda_L - b)})}{(1 + \Delta_L(T_{\text{lim}})) \times (\lambda_L - b)}\right)} \quad \text{for } T_{\text{lim}} < T \leq \tau. \]

(v) For \( T_{\text{lim}} \leq \tau < T \), as in (iii), we obtain (A-11). Since \( T_{\text{lim}} \leq \tau < T \), we get the solution

\[ p(T) = e^{-\lambda_L \times \int_{T}^{\tau} s(x) \, dx + \Delta_L(T_{\text{lim}}) \times (\tau - T) \, \frac{1}{1 + \Delta_L(T_{\text{lim}})}} \times p(\tau) \quad \text{for } T_{\text{lim}} \leq \tau < T, \]

which we again verify by plugging it into (A-11), but now use \( \Delta_L(T_{\text{lim}}) \) from (A-13).

**Clientele Effect**

We prove that for the derived equilibrium prices and constant or monotonically increasing bid-ask spreads \( s(T) \) with \( 0 < s(T) < 1 \), there is a maturity \( T_{\text{lim}} \) such that high-risk investors have no incentive to invest in bonds with longer maturity, i.e.:

**Lemma 1.**
It holds that

\[(A-16) \quad \Delta_H(T) < 0 \quad \text{for all } T \in (T_{\text{lim}}, T_{\text{max}}].\]

Moreover, low-risk investors have no incentive to invest in bonds with maturities shorter than \(T_{\text{lim}}\), i.e.:

**Lemma 2.**

It holds that

\[(A-17) \quad \Delta_L(T) < \Delta_L(T_{\text{lim}}) \quad \text{for all } T \in (0, T_{\text{lim}}).\]

In addition, low-risk investors have no incentive to invest in the money market account, since they hold only bonds, i.e., \(\Delta_L(T) > 0\) for at least one \(T \in (0, T_{\text{max}}].\)

Low-risk investors have higher marginal utility for all bonds than high-risk investors, who have a marginal utility of 0 for bonds with maturity \(T_{\text{lim}}\), therefore \(\Delta_L(T_{\text{lim}}) > \Delta_H(T_{\text{lim}}) = 0\). Hence, the last condition \(\Delta_L(T) > 0\) trivially holds for \(T = T_{\text{lim}}\).

**Proof of Lemma 2:** We verify that \(\Delta_L(T)\) is strictly monotonically increasing in \(T\) for \(T \leq T_{\text{lim}}\) and arbitrary \(T_{\text{lim}}\), i.e., \(\frac{\partial \Delta_L(T)}{\partial T} > 0\): For the case \(T \leq \tau\), \(\Delta_L(T)\) is given as

\[(A-18) \quad \Delta_L(T) = \frac{\lambda_L \times (1 - e^{(\lambda_L-b)\times T})}{(1 - e^{\lambda_L\times T} \times p(T)) \times (\lambda_L - b)} - 1.\]

By employing equation (A-8) for \(p(T)\), using \(0 < b < \lambda_L < \lambda_H\) (see Appendix A.1), and substituting \(b = \lambda_L - c1\) and \(\lambda_H = \lambda_L + c2\) with \(c1, c2 > 0\) and \(c1 < \lambda_L\), the condition \(\frac{\partial \Delta_L(T)}{\partial T} > 0\) simplifies to

\[(A-19) \quad e^{(c1+c2)\times T} \times c1 + c2 > e^{c1\times T} \times (c1 + c2).\]

(A-19) holds for all \(T > 0\) since for \(T = 0\), both sides are equal \((c1 + c2)\), and the first
derivative with respect to $T$ of the left-hand side of (A-19) is larger than that of the right-hand side, i.e.,

$$(c_1 + c_2) \times c_1 \times e^{(c_1+c_2) \times T} > (c_1 + c_2) \times c_1 \times e^{c_1 \times T},$$

which is always true since $c_1, c_2 > 0$.

For the second case with $T > \tau$, rearranging terms and again using $0 < b < \lambda_L < \lambda_H$, the condition $\frac{\partial \Delta_L(T)}{\partial T} > 0$ simplifies to

$$(A-21) \quad (1 - s(T)) \times (e^{T \times \lambda_L} \times p(T) - 1) - \left(1 - e^{(\lambda_L - b) \times \tau}\right) \times \left( -\lambda_L - \frac{\partial p(T)}{\partial T} \right) \frac{1}{(\lambda_L - b)}$$

$$+ \left( \int_{\tau}^{T} e^{-(T-x) \times \lambda_L} \times (1 - s(x)) \times p(x) \, dx \right) \times \left( e^{T \times \lambda_L} \times \lambda_L + e^{T \times \lambda_L} \times \frac{\partial p(T)}{\partial T} \right) > 0.$$ 

We prove that (A-21) holds in two steps: In step (a), we show that (A-21) holds for $T \downarrow \tau$, i.e., we look at the right-side limit of (A-21). In step (b), we show that the first derivative with respect to $T$ of the left-hand side of (A-21) is positive. For (a), rearranging equation (A-2) yields$^{17}$

$$(A-22) \quad s(\tau) = \frac{b \times (e^{(b - \lambda_H) \times \tau} - 1)}{b \times e^{(b - \lambda_H) \times \tau} - \lambda_H}.$$  

$^{17}$Note that for $T > \tau$, we implicitly assume that $\tau$ exists. If $\tau$ does not exist because bid-ask spreads are too large, the already-discussed case for $T \leq \tau$ applies for all $T$. 

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Using again our substitutions \( b = \lambda_L - c_1 \) and \( \lambda_H = \lambda_L + c_2 \) with \( c_1, c_2 > 0 \) and \( c_1 < \lambda_L \), plugging in (A-22) as well as (A-10) for \( p(T) \), we can simplify (A-21) to

\[
(A-23) \quad e^{c_2 \tau} \times c_1 + e^{-c_1 \tau} \times c_2 - (c_1 + c_2) > 0.
\]

Again, it is easy to show that (A-23) holds for all \( \tau > 0 \) by verifying that its left-hand side equals 0 for \( \tau \to 0 \) and its first derivative with respect to \( \tau \) is larger than 0.

For (b), we rearrange (A-21) by employing (A-10) for \( p(T) \) and substituting \( g(T) = T \times \lambda_L - \int_T^T \lambda_H \times s(x) \, dx \) and \( \frac{\partial g(T)}{\partial T} = \lambda_L - \lambda_H \times s(T) \) to finally get

\[
(A-24) \quad \frac{(e^{\theta(T)} \times p(\tau) - 1)}{\lambda_H} \times (\lambda_H - \lambda_L) + \left( \frac{e^{\lambda_L \times T} p(\tau) - 1}{\lambda_H} + \frac{e^{(\lambda_L - b) \times T} - 1}{b - \lambda_L} \right. \\
- \left. \int_T^T \frac{e^{g(x)} \times p(\tau) \times (\lambda_H - \lambda_L)}{\lambda_H} \, dx \right) \times \frac{\partial g(T)}{\partial T} > 0
\]

and it remains to show that the first derivative with respect to \( T \) of the left-hand side of (A-24) is positive:

\[
(A-25) \quad \left( \frac{e^{\lambda_L \times \tau} \times p(\tau) - 1}{\lambda_H} + \frac{e^{(\lambda_L - b) \times \tau} - 1}{b - \lambda_L} \right) - \int_T^T \frac{e^{g(x)} \times p(\tau) \times (\lambda_H - \lambda_L)}{\lambda_H} \, dx \right) \times \frac{\partial^2 g(T)}{\partial T^2} > 0.
\]

As \( \frac{\partial^2 g(T)}{\partial T^2} = -\lambda_H \times \frac{\partial s(T)}{\partial T} \leq 0 \) for monotonically increasing \( s(T) \) and \( -\int_T^T \frac{e^{g(x)} \times p(\tau) \times (\lambda_H - \lambda_L)}{\lambda_H} \, dx < 0 \) (since all factors in the numerator of the integrand are positive), a sufficient condition for (A-25) to hold is that

\[
(A-26) \quad \frac{e^{\lambda_L \times \tau} \times p(\tau) - 1}{\lambda_H} + \frac{e^{(\lambda_L - b) \times \tau} - 1}{b - \lambda_L} < 0.
\]

Using once more our substitutions \( b = \lambda_L - c_1 \) and \( \lambda_H = \lambda_L + c_2 \) with \( c_1, c_2 > 0 \) and
c1 < λ_L and utilizing (A-8) for \( p(\tau) \), (A-26) simplifies to

\[
(A-27) \\
\quad c1 \times (\lambda_L - c1) + e^{c2 \times \tau} \times (c1^2 - c1 \times \lambda_L + (e^{c1 \times \tau} - 1) \times c2 \times (c2 + \lambda_L)) > 0.
\]

As before, it is easy to show that (A-27) holds for all \( \tau > 0 \) by verifying that its left-hand side equals 0 for \( \tau \to 0 \) and its first derivative with respect to \( \tau \) is larger than 0. \( \square \)

**Proof of Lemma 1:** Inequality (A-16) directly follows from \( \frac{\partial \Delta_L(T)}{\partial T} > 0 \) for \( T \leq T_{\text{lim}} \). To see this, assume that for some parameter set \((\lambda_H, \lambda_L, b, T_{\text{max}})\) and given bid-ask spread function \( s(T) \), the wealth of high-risk investors is sufficient to buy all bonds and

the wealth of low-risk investors goes to 0 \( (w_L^* \to 0) \), so that \( T_{\text{lim}}^+ \to T_{\text{max}} \). Suppose now, that for the same parametrization \((\lambda_H, \lambda_L, b, T_{\text{max}})\) and bid-ask spread function \( s(T) \), the wealth of low-risk investors \( w_L^+ \gg 0 \), so that \( T_{\text{lim}}^+ << T_{\text{max}} \). Then it follows with the low-risk investors’ first order condition (A-6) that

\[
(A-28) \\
\quad \Delta_L^+(T) = \Delta_L^+(T_{\text{lim}}^+) \quad \text{for all } T \in (T_{\text{lim}}^+, T_{\text{max}}),
\]

where we use (+) to indicate for which case of \( w^+/w^* \Delta_L(T) \) applies. Moreover, it follows that

\[
(A-29) \\
\quad \Delta_L^+(T_{\text{lim}}^+) = \Delta_L^*(T_{\text{lim}}^+)
\]

as \( p(T_{\text{lim}}^+) \) is not affected by the choice of \( T_{\text{lim}} \geq T_{\text{lim}}^+ \) (dependent on \( \tau \), but independent of \( T_{\text{lim}} \), either equation (A-8) or (A-10) applies for \( p(T) \)). From the fact that \( \frac{\partial \Delta_L(T)}{\partial T} > 0 \) for \( T \leq T_{\text{lim}} \), we directly get

\[
(A-30) \\
\quad \Delta_L^*(T_{\text{lim}}^+) < \Delta_L^*(T) \quad \text{for all } T \in (T_{\text{lim}}^+, T_{\text{lim}}^* = T_{\text{max}}].
\]

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Putting together (A-28)-(A-30), we get

\[(A-31) \quad \Delta^+_L(T) < \Delta^*_L(T) \quad \text{for all } T \in (T^+_\text{lim}, T_{\text{max}}].\]

From the last inequality (A-31), it directly follows that

\[(A-32) \quad p^+(T) > p^*(T) \quad \text{for all } T \in (T^+_\text{lim}, T_{\text{max}}]\]

since lower prices \(p(T)\) directly result in higher marginal utilities due to higher wealth gains. Turning this argument around, we get

\[(A-33) \quad \Delta^+_H(T) < \Delta^*_H(T) \quad \text{for all } T \in (T^+_\text{lim}, T_{\text{max}}].\]

Employing the high-risk investors’ first order condition (A-5)

\[(A-34) \quad \Delta^*_H(T) = 0 \quad \text{for all } T \in (0, T^*_\text{lim} = T_{\text{max}}],\]

it directly follows from (A-33) that

\[(A-35) \quad \Delta^+_H(T) < 0 \quad \text{for all } T \in (T^+_\text{lim}, T_{\text{max}}],\]

which equals inequality (A-16) for \(T_{\text{lim}} = T^+_\text{lim}.\) \(\square\)

This verifies the second part of Proposition 1.

**Appendix A.3 – Market Clearing**

In this section, we verify that markets clear for given equilibrium prices. In doing so, we perform the final step of our iterative approach: we compute a new value of \(T_{\text{lim}}\) for given equilibrium prices.
Total demand for bonds consists of two parts: the demand of reinvesting “old” investors who have not experienced a preference shock, and the demand of new investors. Old investors demand bonds because part of their portfolio has matured and only re-allocate their portfolios. The probability of an old type-i investor not having experienced a preference shock since buying the bond T periods ago equals $e^{-\lambda_i \times T}$. Since bonds are paid back at par, but investors can buy new bonds at the price $p(T)$, old investors absorb a fraction of $\frac{e^{-\lambda_i \times T}}{p(T)}$ of bonds with maturity $T$, and new investors absorb the remainder $1 - \frac{e^{-\lambda_i \times T}}{p(T)}$.

As outlined in Appendix A.2, we focus on the allocation where both high- and low-risk investors hold bonds, and high-risk investors additionally invest in the money market account. In this allocation, markets clear if the wealth of newly arriving investors of both types $w_L + w_H$ suffices to buy all newly issued and prematurely sold bonds that are not absorbed by “old” investors (left inequality), but on the other hand, the wealth of low-risk investors alone does not suffice to buy those bonds (right inequality):

\begin{equation}
\begin{aligned}
&\frac{w_H + w_L}{T_{\text{max}}} > \\
&\int_0^{T_{\text{max}}} p(T) \times \left(1 - \frac{e^{-\lambda(T) \times T}}{p(T)}\right) \times \left[\frac{1}{T} + 1_{\{T > \tau\}} \times \int_{T_{\text{init}}}^{T_{\text{max}}} \frac{1}{T_{\text{init}}} \times \sum_{i=H,L} y_i(T, T_{\text{init}}) \times \lambda_i dT_{\text{init}}\right] dT \\
&> w_L,
\end{aligned}
\end{equation}

with

\begin{equation}
\lambda(T) = \begin{cases} 
\lambda_H, & \text{if } T \leq T_{\text{lim}} \\
\lambda_L, & \text{if } T > T_{\text{lim}}.
\end{cases}
\end{equation}

The term in square brackets in (A-36) gives the total supply of bonds with maturity $T$, which is the sum of newly issued bonds $\frac{1}{T}$ and prematurely sold bonds from both investor types. For that, $y_i(T, T_{\text{init}})$ denotes the fraction of bonds with remaining maturity $T$ and
initial maturity $T_{\text{init}}$ that are held in the portfolios of type-$i$ investors, i.e.,

$$y_L(T, T_{\text{init}}) = \begin{cases} 
0, & \text{if } T, T_{\text{init}} \leq T_{\text{lim}} \\
e^{-\lambda_L \times (T_{\text{lim}} - \max(T, \tau))}, & \text{if } T \leq T_{\text{lim}} \text{ and } T_{\text{init}} > T_{\text{lim}} \\
1, & \text{if } T > T_{\text{lim}}.
\end{cases}$$ 

(A-38)

$$y_H(T, T_{\text{init}}) = 1 - y_L(T, T_{\text{init}}).$$

(A-39)

For bonds with initial maturity $T_{\text{init}} > T_{\text{lim}}$ and current maturity $T \leq T_{\text{lim}}$, a fraction of $e^{-\lambda_L \times (T_{\text{lim}} - \max(T, \tau))}$ is held by old low-risk investors $L$. Bonds with initial and current maturity smaller than $T_{\text{lim}}$ are not held by low-risk investors, bonds with current and initial maturity larger than $T_{\text{lim}}$ are only held by low-risk investors.

The right inequality of (A-36) is automatically satisfied if the condition we derive below to determine $T_{\text{lim}}$ yields a $T_{\text{lim}} \in (0, T_{\text{max}})$. By inserting the formulas for $p(T)$ from Appendix A.2 for a given parameter set, it is easy to verify the left inequality of (A-36).\textsuperscript{18}

To compute a $T_{\text{lim}}$ consistent with equilibrium prices, we exploit the market clearing condition for bonds with maturities $T_{\text{init}} \in (T_{\text{lim}}, T_{\text{max}}]$ that are held by low-risk investors. In analogy to (A-36), we solve

$$w_L = \int_{T_{\text{lim}}}^{T_{\text{max}}} p(T) \times \left(1 - \frac{e^{-\lambda_L \times T}}{p(T)}\right) \times \left[\frac{1}{T} \times \frac{1}{T_{\text{init}}} \times \lambda_L \times dT_{\text{init}}\right] dT$$

for $T_{\text{lim}}$. To illustrate the determination of $T_{\text{lim}}$, consider the extreme case of $w_L \to 0$. As

\textsuperscript{18}If low-risk investors’ wealth alone is sufficient to buy all bonds, i.e., $T_{\text{lim}} = 0$, markets clear if $w_L > \int_{T_{\text{lim}}}^{T_{\text{max}}} p(T) \times \left(1 - \frac{e^{-\lambda_L \times T}}{p(T)}\right) \times \left[\frac{1}{T} \times \int_{T_{\text{init}}}^{T_{\text{max}}} \frac{1}{T_{\text{init}}} \times \lambda_L \frac{dT_{\text{init}}}{dT}\right] dT$. However, this case is less interesting as high-risk investors do not play a role. We do not consider the degenerate allocation where $w_H + w_L$ is exactly equal to $\int_{0}^{T_{\text{max}}} p(T) \times \left(1 - \frac{e^{-\lambda_T \times T}}{p(T)}\right) \times \left[\frac{1}{T} \times \int_{T_{\text{init}}}^{T_{\text{max}}} \frac{1}{T_{\text{init}}} \times \lambda_L \times \sum_{i=H, L} y_i(T, T_{\text{init}}) \times \lambda_i \times dT_{\text{init}}\right] dT$. In this case, bond prices would primarily reflect the economy’s wealth constraint and strongly depend on wealth, which is hard to quantify empirically.
the integrand of the outer integral in equation (A-40) is strictly positive, $T_{hm} \rightarrow T_{max}$.

**Appendix B: Data Selection Procedure**

We collect the reporting date and time, the transaction yield, price, volume, and the information whether a trade is an interdealer trade or a customer buy or sell trade from Enhanced TRACE. Our observation period runs from the full implementation of TRACE in Oct. 1, 2004 until Sept. 30, 2012.\(^{19}\) We start with filtering out erroneous trades as described in Dick-Nielsen (2009) and Dick-Nielsen (2014). For the remaining bonds, we collect information on the bond’s maturity, coupon, and other features from Reuters and Bloomberg using the bond’s Committee on Uniform Security Identification Procedures (CUSIP). We drop all bonds which are not plain vanilla fixed rate bonds without any extra rights. We also collect the rating history from Reuters and drop all observations for bonds on days on which fewer than two rating agencies (Standard&Poor’s, Moody’s, Fitch) report an investment-grade rating. We exclude private placements, bonds with more than 30 years remaining maturity, and all bonds that are not classified as senior unsecured in the Markit data base.

For the sample used in the analysis of liquidity premia, we follow Dick-Nielsen (2009) and additionally drop all transactions with nonstandard trade or settlement conditions. Moreover, we exclude all trades for which the yield calculated from the reported price does not exactly match the reported yield (less than 1% of the trades). For the turnover analysis and as a control variable, we collect the history of outstanding notional amounts for each bond from Reuters. We use Treasury yields as the risk-free interest rate curve and employ swap rates instead as a robustness check in the Internet Appendix 5. For the par Treasury yield curve, we use updated data from Gürkaynak, Sack, and Wright (2007) pub-

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\(^{19}\)Since Enhanced TRACE contains information that has previously not been disseminated to the public, it is only available with a lag of 18 months.
lished on the Federal Reserve's web site (www.federalreserve.gov). We use U.S.-Dollar swap curves available via Bloomberg for maturities larger than 3 months and extend the curve at the short end by linearly interpolating the 6-months rate with U.S.-Dollar London Interbank Offered Rates (LIBOR) for a maturity of 1 and 3 months. We also account for the different day count conventions in swap and LIBOR markets. Table B1 summarizes the data selection procedure and the number of observations for our final sample and the subsamples used in our robustness checks in the Internet Appendix 5.

Insert Table B1 about here.
Figure 1: **Turnover – Hump-Shaped Trading Volume and Aging Effect**

Figure 1 presents secondary-market turnover for the baseline case where the rate $\lambda$ at which preference shocks occur equals 0.5 for high-risk investors and 0.25 for low-risk investors, the time preference rate increases from 0 to $b = 0.02$ if a preference shock occurs, bid-ask spreads $s$ equal 0.3%, the maximum bond maturity $T_{\text{max}}$ equals 10 years, both investor types enter the economy so that total wealth from both types arrives with $w_H = 4$ and $w_L = 2$. In the resulting equilibrium allocation, high-risk investors invest in bonds with maturities up to $T_{\text{lim, baseline}} = 1.462$ years, and only bonds with a maturity higher than $\tau_{\text{baseline}} = 0.156$ years are sold if a preference shock occurs. The dotted line represents turnover of bonds with initial maturity $T_{\text{init}} < T_{\text{lim}}$, the dashed line depicts turnover of bonds with initial maturity $T_{\text{init}} > T_{\text{lim}}$, and the solid line aggregates turnover over all bonds.
Figure 2: Liquidity Premia and Spillover Effect

Figure 2 presents liquidity premia for the baseline case (thick lines) where the rate $\lambda$ at which preference shocks occur equals 0.5 for high-risk investors and 0.25 for low-risk investors, the time preference rate increases from 0 to $b = 0.02$ if a preference shock occurs, bid-ask spreads $s$ equal 0.3% for all maturities, the maximum bond maturity $T_{\text{max}}$ equals 10 years, both investor types enter the economy so that total wealth from both types arrives with $w_H = 4$ and $w_L = 2$. In the resulting equilibrium allocation, high-risk investors invest in bonds with maturities up to $T_{\text{lim, baseline}} = 1.462$ years, and only bonds with a maturity higher than $\tau_{\text{baseline}} = 0.156$ years are sold if a preference shock occurs. Thin lines present liquidity premia for the case of higher liquidity demand for high-risk investors ($\lambda_H = 1.0$). All other parameters are identical to the baseline case. For this specification, critical maturities $\tau_{\lambda_H=1.0} = 0.163$ and $T_{\text{lim}, \lambda_H=1.0} = 1.405$ only change marginally compared to the baseline specification. Solid lines depict $\text{illiq}^{\text{ask}}(T)$, dotted lines $\text{illiq}^{\text{mid}}(T)$, and dashed lines $\text{illiq}^{\text{bid}}(T)$.

![Graph showing liquidity premia](image-url)
Figure 3: Liquidity Premia – Increasing Bid-Ask Spreads and Spillover Effect

Figure 3 displays liquidity premia for the case where we have calibrated bid-ask spreads to observed prices in thick lines \( s(T) = 0.0044 + 0.0241 (1 - e^{-0.1014T}) \). The rate \( \lambda \) at which preference shocks occur equals 0.5 for high-risk investors and 0.25 for low-risk investors, the time preference rate increases from 0 to \( b = 0.02 \) if a preference shock occurs, the maximum bond maturity \( T_{\text{max}} \) equals 10 years, both investor types enter the economy so that total wealth from both types arrives with \( w_H = 4 \) and \( w_L = 2 \). In the resulting equilibrium allocation, high-risk investors invest in bonds with maturities up to \( T_{\text{lim, calibrated}} = 1.368 \) years, and only bonds with a maturity higher than \( \tau_{\text{calibrated}} = 0.270 \) years are sold if a preference shock occurs. Thin lines present liquidity premia for the case of higher short-term bid-ask spreads \( s_{\text{higher}}(T) = s(T) \) for \( T < 1.25 \) and \( s_{\text{higher}}(T) = s(T(1.25)) \) for \( T \geq 1.25 \). All other parameters are identical to the baseline case. For this specification, critical maturities are now \( \tau_{\text{higher}} = 0.402 \) and \( T_{\text{lim, higher}} = 1.359 \). Solid lines depict \( \text{illiq}^{\text{ask}}(T) \), dotted lines \( \text{illiq}^{\text{mid}}(T) \), and dashed lines \( \text{illiq}^{\text{bid}}(T) \).
Figure 4: **Empirical term structures of ask and bid liquidity premia**

Figure 4 presents the average term structures of ask and bid liquidity premia together with the predictions of our model (see Figure 3). In Graph A, the liquidity premium $\text{illiq}^{\text{ask/bid}}_\text{diff}$ is determined as the difference of the bond yield and a theoretical credit adjusted yield calculated by discounting the bond’s cash flows with a bootstrapped discount curve computed from Treasury yields and a CDS curve. In Graph B, the liquidity premium $\text{illiq}^{\text{ask/bid}}_\text{reg}$ is determined as in Dick-Nielsen et al. (2012) as the yield spread proportion explained by the liquidity measure $lm$ in a linear regression, where $lm$ is the equal-weighted average of the Amihud (2002) liquidity measure, imputed roundtrip costs as in Feldhütter (2012), and the standard deviations of these measures. Squares indicate average ask liquidity premia, circles show average bid liquidity premia. The solid line depicts model implied $\text{illiq}^{\text{ask}}(T)$, the dashed line shows model implied $\text{illiq}^{\text{bid}}(T)$. The sample period is from Oct. 1, 2004 to Sept. 30, 2012.
Table 1: Regression of Ask and Bid Liquidity Premia on Duration

Table 1 presents the regression of ask and bid liquidity premia (in percentage points) on the bond’s duration and control variables for different breakpoints that separate the short end from longer maturities of the liquidity term structure:

\[
\begin{align*}
iliq^{\text{ask/diff/reg}}(T) &= \alpha^{\text{ask}} + \beta_1^{\text{ask}} I_{T \leq \theta} \times (T - \theta) + \beta_2^{\text{ask}} I_{T > \theta} \times (T - \theta) + \gamma^{\text{ask}} \text{CONTROLS} + \varepsilon, \\
iliq^{\text{bid/diff/reg}}(T) &= \alpha^{\text{bid}} + \beta_1^{\text{bid}} I_{T \leq \theta} \times (T - \theta) + \beta_2^{\text{bid}} I_{T > \theta} \times (T - \theta) + \gamma^{\text{bid}} \text{CONTROLS} + \varepsilon.
\end{align*}
\]

In Panel A (panel regression), the liquidity premium \(\text{illiq}^{\text{ask/bid}}\) is determined for each bond and each trade as the difference of the bond yield and a theoretical credit adjusted yield calculated by discounting the bond’s cash flows with a bootstrapped discount curve computed from Treasury yields and a CDS curve. In Panel B (cross-sectional regression), the average liquidity premium \(\text{illiq}^{\text{ask/bid}}\) for each monthly duration bucket is determined as in Dick-Nielsen et al. (2012) as the proportion of the yield spread (in excess of the Treasury yield curve) explained by the liquidity measure \(ln\) in a linear regression, where \(ln\) is the equal-weighted average of the Anilud (2002) liquidity measure, imputed roundtrip costs as in Feldhüter (2012), and the standard deviations of these measures. The explanatory variable is the duration \(T\) (in years) minus the breakpoint \(\theta\) for \(T \leq \theta\) and \(T > \theta\). In Panel A, we additionally include the control variables AGE in years, the average numerical rating (RATING), and the logarithm of the outstanding amount (\(\ln(\text{AMT})\)) and use firm and month fixed effects. The breakpoints \(\theta\) equal 3 months, 6 months, 1 year, 2 years, and 3 years and we estimate an endogenous breakpoint \(\theta^*\). In Panel A, we present standard errors clustered at the firm level in parentheses. In Panel B, we use White (1982) standard errors. The sample period is from Oct. 1, 2004 to Sept. 30, 2012. * and ** indicate significance at the 5% and 1% levels, respectively.

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<tr>
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<th>Bid</th>
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<tr>
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<td>(\theta = 0.5)</td>
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<td>3.3402**</td>
<td>1.4416**</td>
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<td>(1.2911)</td>
<td>(0.4988)</td>
<td>(0.2050)</td>
</tr>
<tr>
<td>(I_{(T &gt; \theta)} \times (T - \theta))</td>
<td>0.0573**</td>
<td>0.0550**</td>
</tr>
<tr>
<td>(0.0110)</td>
<td>(0.0112)</td>
<td>(0.0113)</td>
</tr>
</tbody>
</table>

| Controls | | |
| AGE [in years] | 0.0004 | 0.0015 | 0.0056 | 0.0125 | 0.0149 | 0.0142 | 0.0284** | 0.0266** | 0.0251** | 0.0252** | 0.0260** | 0.0265** |
| (0.0070) | (0.0070) | (0.0070) | (0.0070) | (0.0081) | (0.0081) | (0.0081) | (0.0057) | (0.0055) | (0.0053) | (0.0053) | (0.0050) | (0.0055) |
| RATING | -0.0139 | -0.0118 | -0.0069 | 0.0010 | 0.0014 | 0.0015 | 0.1665** | 0.1657** | 0.1665** | 0.1674** | 0.1676** | 0.1657** |
| (0.0487) | (0.0484) | (0.0481) | (0.0472) | (0.0470) | (0.0471) | (0.0542) | (0.0538) | (0.0537) | (0.0539) | (0.0542) | (0.0538) |
| \(\ln(\text{AMT})\) | 0.0586 | 0.0584 | 0.0591 | 0.0584 | 0.0515 | 0.0553 | -0.0429** | -0.0419 | -0.0420 | -0.0436** | -0.0433** | -0.0418 |
| (0.0321) | (0.0318) | (0.0316) | (0.0308) | (0.0295) | (0.0306) | (0.0217) | (0.0221) | (0.0221) | (0.0219) | (0.0214) | (0.0221) |

| Firm Fixed Effects | Yes | Yes |
| Month Fixed Effects | Yes | Yes |
| \(I_{(T > \theta)} \times (T - \theta)\) | -3.2830* | -1.3886** | -0.8549** | -0.4527** | -0.2826** | -0.3487** | 8.7601** | 2.5763** | 0.7902** | 0.1736** | 0.0797** | 2.4045** |
| (1.2943) | (0.5030) | (0.2097) | (0.1042) | (0.0639) | (0.0800) | (0.6088) | (0.2546) | (0.1101) | (0.0544) | (0.0371) | (0.2416) |
| \(I_{(T \leq \theta)} \times (T - \theta)\) | -1.9623 | 3.543,343 |
| N | 1,962,313 | |
| \(R^2\) | 0.3934 | 0.3946 | 0.4003 | 0.4094 | 0.4099 | 0.4102 | 0.3877 | 0.3907 | 0.3872 | 0.3815 | 0.3796 | 0.3908 |
### Table: Panel B: Liquidity Premium illiq

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<td>(0.0300)</td>
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<tr>
<td>θ = 0.5</td>
<td>0.1673**</td>
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<td>(0.0300)</td>
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<tr>
<td>θ = 1</td>
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<tr>
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<td>(0.0281)</td>
<td>(0.0281)</td>
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<tr>
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<td>0.2963**</td>
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<td>(0.0281)</td>
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<td>(0.0281)</td>
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<tr>
<td>θ = 0.25</td>
<td>0.5736**</td>
<td>0.5736**</td>
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<tr>
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<td>(0.0315)</td>
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<tr>
<td>θ = 0.5</td>
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<td>0.5599**</td>
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<tr>
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<td>(0.0300)</td>
<td>(0.0300)</td>
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<tr>
<td>θ = 1</td>
<td>0.5844**</td>
<td>0.5844**</td>
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<tr>
<td></td>
<td>(0.0309)</td>
<td>(0.0309)</td>
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<tr>
<td>θ = 2</td>
<td>0.5886**</td>
<td>0.5886**</td>
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<tr>
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<td>(0.0312)</td>
<td>(0.0312)</td>
</tr>
<tr>
<td>θ = 3</td>
<td>0.6277**</td>
<td>0.6277**</td>
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<tr>
<td></td>
<td>(0.0311)</td>
<td>(0.0311)</td>
</tr>
<tr>
<td>θ* = 0.5000</td>
<td>0.5599**</td>
<td>0.5599**</td>
</tr>
<tr>
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<td>(0.0300)</td>
<td>(0.0300)</td>
</tr>
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**Notes:**
- **N:** 221
- **R²:** 0.2649

- **T≤θ** × (T − θ)
  - | μ  | (0.0305) | (0.0305) | (0.0305) | (0.0305) | (0.0305) | (0.0305) | (0.0305) | (0.0305) | (0.0305) | (0.0305) | (0.0305) |
    - | -1.0185 | -0.5062* | -0.3072** | -0.2075** | -0.1571** | -0.0903** | 5.0268** | 1.7205** | 0.4554** | 0.0449 | -0.0296 | 1.7205** |
    - | (0.6822) | (0.2098) | (0.0812) | (0.0362) | (0.0220) | (0.0125) | (0.2854) | (0.2871) | (0.1545) | (0.0736) | (0.0438) | (0.2871) |

- **T>θ** × (T − θ)
  - | μ  | (0.0035) | (0.0036) | (0.0037) | (0.0038) | (0.0040) | (0.0052) | (0.0039) | (0.0039) | (0.0041) | (0.0044) | (0.0047) | (0.0039) |
    - | -1.0185 | -0.5062* | -0.3072** | -0.2075** | -0.1571** | -0.0903** | 5.0268** | 1.7205** | 0.4554** | 0.0449 | -0.0296 | 1.7205** |
    - | (0.6822) | (0.2098) | (0.0812) | (0.0362) | (0.0220) | (0.0125) | (0.2854) | (0.2871) | (0.1545) | (0.0736) | (0.0438) | (0.2871) |
Table 2: Regression of Turnover on Duration and Age

Table 2 presents the panel regression of turnover for the two subsamples on duration, age, and control variables for different breakpoints:

$$ \text{turnover}(T_{j,t}) = \alpha + \beta_1 I_{\{T_{j,t} \leq \theta\}} \times (T_{j,t} - \theta) + \beta_2 I_{\{T_{j,t} > \theta\}} \times (T_{j,t} - \theta) + \beta_3 \text{AGE}_{j,t} + \gamma \text{CONTROLS}_{j,t} + \varepsilon_{j,t}, $$

where turnover($T$) is calculated as the average daily turnover for each bond and each calendar month. The left panel contains the regression results for the full sample, the right panel contains the regression results for the subsample that excludes bonds 2 months prior to and 6 months after changes in their outstanding amount. The explanatory variables are the bond’s duration $T$ (in years) minus the breakpoint $\theta$ for $T \leq \theta$ and $T > \theta$ as well as $\text{AGE}$ (in years). The control variables are the average numerical rating ($\text{RATING}$) and the logarithm of the outstanding amount (ln($\text{AMT}$)). The breakpoints $\theta$ are given by 3 months, 6 months, 1 year, 2 years, and 3 years and we estimate an endogenous breakpoint $\theta^*$. We use month fixed effects. Clustered standard errors at the firm level are presented in parentheses. Parameter estimates and standard errors are multiplied by 1,000. The sample period is from Oct. 1, 2004 to Sept. 30, 2012. * and ** indicate significance at the 5% and 1% levels, respectively.

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<td>0.0425*</td>
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<tr>
<td>ln(AMT)</td>
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<td>0.4310**</td>
</tr>
<tr>
<td>Firm Fixed Effects</td>
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<td>Yes</td>
</tr>
<tr>
<td>Month Fixed Effects</td>
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<td></td>
</tr>
<tr>
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<tr>
<td>R$^2$</td>
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</table>
Table 3: Spillover Analysis

Table 3 presents a vector autoregression (VAR) analysis of monthly average ask and bid liquidity premia (in percentage points) on lagged liquidity premia for different breakpoints that separate short-term from long-term maturities of the liquidity term structure:

\[
\text{illiq}_t^{\text{ask}}(T < \theta) = \alpha_{\text{short}}^{\text{ask}} + \sum_{i=1}^{2} \phi_{i,\text{short}}^{\text{ask}} \text{illiq}_{t-i}^{\text{ask}}(T < \theta) + \sum_{i=1}^{2} \beta_{i,\text{long}}^{\text{ask}} \text{illiq}_{t-i}^{\text{ask}}(T \geq \theta) + \varepsilon_t,
\]

\[
\text{illiq}_t^{\text{bid}}(T < \theta) = \alpha_{\text{short}}^{\text{bid}} + \sum_{i=1}^{2} \phi_{i,\text{short}}^{\text{bid}} \text{illiq}_{t-i}^{\text{bid}}(T < \theta) + \sum_{i=1}^{2} \beta_{i,\text{long}}^{\text{bid}} \text{illiq}_{t-i}^{\text{bid}}(T \geq \theta) + \varepsilon_t,
\]

\[
\text{illiq}_t^{\text{ask}}(T \geq \theta) = \alpha_{\text{long}}^{\text{ask}} + \sum_{i=1}^{2} \phi_{i,\text{short}}^{\text{ask}} \text{illiq}_{t-i}^{\text{ask}}(T < \theta) + \sum_{i=1}^{2} \phi_{i,\text{long}}^{\text{ask}} \text{illiq}_{t-i}^{\text{ask}}(T \geq \theta) + \varepsilon_t,
\]

\[
\text{illiq}_t^{\text{bid}}(T \geq \theta) = \alpha_{\text{long}}^{\text{bid}} + \sum_{i=1}^{2} \phi_{i,\text{short}}^{\text{bid}} \text{illiq}_{t-i}^{\text{bid}}(T < \theta) + \sum_{i=1}^{2} \phi_{i,\text{long}}^{\text{bid}} \text{illiq}_{t-i}^{\text{bid}}(T \geq \theta) + \varepsilon_t.
\]

Average monthly liquidity premia \( \text{illiq}_{t}^{\text{ask/bid}} \) for all bonds with durations above and below breakpoint \( \theta \) are determined with the difference approach, i.e., as the difference of the bond yield and a theoretical credit adjusted yield calculated by discounting the bond’s cash flows with a bootstrapped discount curve computed from Treasury yields and a CDS curve. A time trend and the square of the time are removed from the time series of monthly average liquidity premia. The breakpoints \( \theta \) equal 3 months, 6 months, 1 year, 2 years, and 3 years. We present Newey and West (1987) standard errors with 3 lags in parentheses. We provide \( \chi^2 \) statistics for the null hypotheses, that i) both lag parameters are jointly 0 and ii) the sum of both lag parameters is 0. The sample period is from Oct. 1, 2004 to Sept. 30, 2012. * and ** indicate significance at the 5% and 1% levels, respectively.

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<th>( \theta = 1 )</th>
<th>( \theta = 2 )</th>
<th>( \theta = 3 )</th>
<th>( \theta = 0.25 )</th>
<th>( \theta = 0.5 )</th>
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<td>-0.0026</td>
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<td>-0.0021</td>
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<td>(0.0833)</td>
<td>(0.0941)</td>
<td>(0.0832)</td>
<td>(0.0727)</td>
<td>(0.0640)</td>
<td>(0.1318)</td>
<td>(0.1311)</td>
<td>(0.1301)</td>
<td>(0.1075)</td>
</tr>
<tr>
<td>\text{illiq}_{t-1}^{\text{ask/bid}}(T &lt; \theta)</td>
<td>0.5401**</td>
<td>0.6882**</td>
<td>0.7016**</td>
<td>0.5653**</td>
<td>0.5581**</td>
<td>0.7091**</td>
<td>0.7716**</td>
<td>0.7531**</td>
<td>0.5885**</td>
</tr>
<tr>
<td></td>
<td>(0.1514)</td>
<td>(0.0905)</td>
<td>(0.1051)</td>
<td>(0.0591)</td>
<td>(0.0545)</td>
<td>(0.1509)</td>
<td>(0.1410)</td>
<td>(0.1106)</td>
<td>(0.0890)</td>
</tr>
<tr>
<td>\text{illiq}_{t-2}^{\text{ask/bid}}(T &lt; \theta)</td>
<td>0.0174</td>
<td>0.0464</td>
<td>0.0091</td>
<td>0.0788*</td>
<td>0.0969</td>
<td>-0.2045</td>
<td>-0.1338</td>
<td>-0.1065</td>
<td>-0.0018</td>
</tr>
<tr>
<td></td>
<td>(0.1018)</td>
<td>(0.0848)</td>
<td>(0.0450)</td>
<td>(0.0571)</td>
<td>(0.1303)</td>
<td>(0.1416)</td>
<td>(0.0687)</td>
<td>(0.1106)</td>
<td>(0.1375)</td>
</tr>
<tr>
<td>\text{illiq}_{t-1}^{\text{ask/bid}}(T \geq \theta)</td>
<td>0.1368</td>
<td>-0.1090</td>
<td>-0.7453</td>
<td>0.0378</td>
<td>0.2667</td>
<td>0.4907</td>
<td>-0.9417</td>
<td>-1.1710</td>
<td>-0.4111</td>
</tr>
<tr>
<td></td>
<td>(0.8303)</td>
<td>(0.8591)</td>
<td>(0.8525)</td>
<td>(0.9329)</td>
<td>(0.7984)</td>
<td>(2.0215)</td>
<td>(1.4548)</td>
<td>(1.6039)</td>
<td>(1.6697)</td>
</tr>
<tr>
<td>\text{illiq}_{t-2}^{\text{ask/bid}}(T \geq \theta)</td>
<td>-0.4140</td>
<td>0.3669</td>
<td>0.0488</td>
<td>-0.3179</td>
<td>-0.4224</td>
<td>-0.0422</td>
<td>0.8564</td>
<td>0.9505</td>
<td>0.5810</td>
</tr>
<tr>
<td></td>
<td>(0.6865)</td>
<td>(0.6038)</td>
<td>(0.6631)</td>
<td>(0.6524)</td>
<td>(0.5320)</td>
<td>(1.5100)</td>
<td>(0.9852)</td>
<td>(1.9067)</td>
<td>(1.0817)</td>
</tr>
</tbody>
</table>

**H0:** \( \phi_{\text{ask/bid}}^{\text{short}} = 0, \phi_{\text{ask/bid}}^{\text{long}} = 0 \)

**H1:** \( \phi_{\text{ask/bid}}^{\text{short}} + \phi_{\text{ask/bid}}^{\text{long}} = 0 \)

**H2:** \( \beta_{\text{long}}^{\text{ask/bid}} = 0, \beta_{\text{long}}^{\text{ask/bid}} = 0 \)

**H3:** \( \beta_{\text{long}}^{\text{ask/bid}} + \beta_{\text{long}}^{\text{ask/bid}} = 0 \)

N = 94

\( R^2 \) = 0.3060 0.4020 0.4450 0.3660 0.3927 0.4535 0.3928 0.3775 0.3428 0.3974
<table>
<thead>
<tr>
<th></th>
<th>Ask</th>
<th>Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>θ = 0.25</td>
<td>θ = 0.5</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0001</td>
<td>-0.0004</td>
</tr>
<tr>
<td></td>
<td>(0.0161)</td>
<td>(0.0148)</td>
</tr>
<tr>
<td>illiq_{ask/bid} (T &lt; θ)</td>
<td>-0.0124 (0.0246)</td>
<td>0.0317 (0.0207)</td>
</tr>
<tr>
<td>illiq_{ask/bid} (T ≥ θ)</td>
<td>0.0328 (0.0205)</td>
<td>0.0027 (0.0129)</td>
</tr>
<tr>
<td>illiq_{ask/bid} (T ≥ θ)</td>
<td>-0.1198 (0.0677)</td>
<td>0.0347 (0.0905)</td>
</tr>
<tr>
<td>H₀: β_{ask/bid} + β_{ask/bid} = 0</td>
<td>180.9102** (248.1863**)</td>
<td>232.8914** (372.3914**)</td>
</tr>
<tr>
<td>N</td>
<td>94</td>
<td>94</td>
</tr>
<tr>
<td>R²</td>
<td>0.7267</td>
<td>0.7515</td>
</tr>
</tbody>
</table>
Table B1: Sample Description

Table B1 presents the procedure used to arrive at the final samples employed in our main analysis and in the robustness checks in the Internet Appendix 5, the number of trades, the number of bonds, and the traded notional value in billion U.S.-Dollar. The last column shows the number of bond-month observations used for the calculation of liquidity premia illiqask/bidreg and with the regression-based approach. The sample period is from Oct. 1, 2004 to Sept. 30, 2012.

<table>
<thead>
<tr>
<th>Number of Trades</th>
<th>Number of Bonds</th>
<th>Traded Notional Value (in bn. USD)</th>
<th>Number of Bond-Month Observations to Calculate illiqask/bidreg</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Trade Entries Within the TRACE Database</td>
<td>92,188,862</td>
<td>77,003</td>
<td>86,842</td>
</tr>
<tr>
<td>Subtotal after filtering out erroneous and duplicate trade entries with the procedures described in Dick-Nielsen (2009, 2014)</td>
<td>57,806,924</td>
<td>63,829</td>
<td>38,376</td>
</tr>
<tr>
<td>Turnover sample: excluding bonds with missing information (in Bloomberg, Reuters, or Markit), bonds with embedded call or put options (incl. make-whole call provisions, death puts, poison puts, ...), bonds with remaining time to maturity of more than 30 years, bonds with sinking funds, zero coupon bonds, convertible bonds, bonds with variable coupon payments, bonds with other non-standard cash flow or coupon structures, issues which do not have an investment grade rating from at least two rating agencies (i.e., Moody’s, S&amp;P, or Fitch) at the trading date, bonds which are not classified as senior unsecured, private placements, bonds with government guarantee, trades on days for which a Treasury curve is not available, trades that could not be matched to CDS data</td>
<td>10,483,321</td>
<td>2,786</td>
<td>4,783</td>
</tr>
</tbody>
</table>

Samples used to calculate liquidity premia: in addition to the turnover sample, we exclude interdealer trades, trades under non-standard terms (e.g., special settlement or sale conditions), trades for which we could not replicate the reported yield from the trade price, and bonds with durations of less than one month

<table>
<thead>
<tr>
<th>Main sample:</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Trades for which dealer is seller (ask)</td>
<td>3,543,343</td>
<td>2,637</td>
<td>1,516</td>
</tr>
<tr>
<td>Trades for which dealer is buyer (bid)</td>
<td>1,962,313</td>
<td>2,631</td>
<td>1,479</td>
</tr>
</tbody>
</table>

Swap-implied liquidity premia: in contrast to main sample excluding trades on days without an available swap curve (instead of Treasury curve)

| Trades for which dealer is seller (ask) | 3,482,571 | 2,636 | 1,495 | 62,207 |
| Trades for which dealer is buyer (bid) | 1,926,684 | 2,629 | 1,461 | 61,259 |

AAA bonds before financial crisis: in contrast to main sample not matched to CDS data, only trades until March 31, 2007 for which the bond is rated AAA from at least two rating agencies (i.e., Moody’s, S&P, or Fitch) at the trading day

| Trades for which dealer is seller (ask) | 116,404 | 163 | 49 | 2,025 |
| Trades for which dealer is buyer (bid) | 66,449 | 161 | 48 | 2,008 |