Zero Crossing

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Abstract

We show that ex ante lower bound beliefs embedded in interest rate option prices both in the US and the Eurozone are inconsistent with a zero lower bound on nominal interest rates. In contrast, a model-free analysis of option-implied moments of short-term interest-rates during the past decade of ultra-low interest rates suggests a substantial downward revision of lower bound beliefs. Before Eurozone interest rates turned negative, derivatives markets both in the Eurozone and the US were consistent with a lower bound around zero: highly right-skewed distributions, high kurtosis, and a strong level-dependence of volatility at low interest rates. According to our structural break regressions, interest rate distributions show much weaker signs of being close to any lower bound precisely from the time when interest rates fell below zero in many economies in 2014/2015.

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1 Introduction

It has long been a strong consensus among academics and practitioners alike that nominal interest rates cannot become negative granted by the opportunity of people to hold currency at presumably negligible cost. The zero lower bound was therefore widely believed to be a major driver for the shape and evolution of the yield curve and macroeconomic dynamics during past episodes of near-zero interest rates (see e.g. Bauer and Rudebusch (2016); King (2019); Gust, Herbst, López-Salido, and Smith (2017)). Consequently, considerable effort has been made towards models with the ability to incorporate a lower bound constraint (Krippner (2012); Wu and Xia (2016); Filipović, Larsson, and Trolle (2017)). Extended periods of deeply negative interest rates in numerous major economies, however, proved the assumption of strictly positive nominal interest rates wrong, possibly calling the central economic role of a lower bound into serious question.

This paper analyzes *ex ante* beliefs about a lower bound on interest rates through time. The hindsight knowledge that interest rates in fact can become negative tells little about how lower bound beliefs might have impacted economic expectations and interest-rate distributions. The crucial question we ask is how perceptions of a lower bound at the time shaped market expectations. Market participants may in fact have always correctly anticipated that interest rates can very well become (substantially) negative. In this case, the (zero) lower bound should have been largely irrelevant as it would not have entered market expectations. But it is also well conceivable that markets have been caught by surprise by this unprecedented event and that beliefs about a lower bound therefore changed over time. In this case, the zero lower bound may have had an important effect at the time investors were still convinced of the non-negativity constraint. In both cases, it is unclear how much a potentially negative lower bound impacts expectations in a negative interest rate environment. Finally, investors in the US may still hold on to their zero lower bound belief despite opposing international evidence. At least, this is a common implicit assumption in the literature focusing on US markets, where the breach of this bound in other economies has been largely ignored so far.

To address these questions, we study lower bound beliefs embedded in interest rate option prices in both the US and the Eurozone. Options reflect forward-looking distributions of market beliefs and hence allow us to directly estimate the conditional impact of a lower bound on this distribution. So far, the literature studying the impact of a lower bound on interest rate option prices finds conflicting evidence. Studies focusing only on the period prior to the violation of the zero lower bound document option price dynamics consistent with a strong impact of such a positivity constraint on interest rate expectations (Mertens and Williams (2021); Filipović et al. (2017)). Bauer and Chernov (2022), however, conclude that option-implied distributions are little affected by a lower bound as they behave very different during various periods of low interest rates. We argue this seemingly conflicting evidence can be explained by a substantial change in lower bound beliefs when interest rates became negative in 2014/2015. In fact, our results suggest that option markets initially did not expect the possibility of (substantially) negative interest rates. Accordingly, implied distributions for near-zero interest rates exhibited clear evidence of a zero lower bound for several years. When markets witnessed the violation of the zero lower bound in the Eurozone and other economies, our data points to a strong revision of lower bound beliefs both in the Eurozone and the US. Consequently, (option-implied) distributions were no longer affected by a lower bound constraint.

We use three measures to extract lower bound beliefs from option prices. In a model-free theoretical analysis we show how the higher moments of (option-implied) interest-rate distributions can be used to infer whether interest rates are expected to be close to a lower bound. Particularly, we show that interest rate skewness and kurtosis are necessarily high and positive when distributions are tightly constrained by a lower bound. Furthermore, we follow Filipović et al. (2017) and Kim and Singleton (2012) and measure the conditional level-volatility dependence as an indicator for lower bound constrained interest rates. Close to the lower bound there should be a tight relation between the expected interest rate level and the volatility of interest-rate distributions. Together these measures provide us with a comprehensive description of interest rates distribution characteristics at the lower bound. We empirically analyze these distribution characteristics with model-free option-implied moments of short-term interest rates.

Consistent with the anticipation of a zero lower bound, the time-series of all three measures suggest tightly constrained interest-rate distributions during the near-zero interest rate periods both in the US and the Eurozone until 2014/2015. Interest rate skewness, kurtosis, and level-dependence of volatility are all highly elevated and implied moments are theoretically in line with a market-anticipated lower bound at zero. Around 2014/2015, however, all indications of lower-bound constrained distributions experience a sharp drop despite decreasing interest rates in the Eurozone and continued low interest rates in the US. Interest-rate skewness and kurtosis imply a (deeply) negative market anticipated lower bound after 2015 and hence much less constrained interest rates. These results provide first evidence against the hypothesis that markets always anticipated the possibility of negative interest rates, but rather suggest a substantial change in lower-bound beliefs.

To test this mechanism more formally, we proceed in two steps. First, we examine whether interest-rate levels alone are able to explain time-variations in measures for lower-bound constrained distributions. Indeed this is not the case. Next, we formally investigate whether and when market lower bound beliefs may have changed over time. To this end we examine level-conditional distribution characteristics in a structural-break regression framework. Hence, we test whether a potentially lower bound induced level-conditional impact on distributions experienced a structural change because of a (downward) shift in lower bound beliefs. We find that low interest rates lead to strongly lower bound shaped distributions only in the former part of our sample, both in the Eurozone and the US. That is, we detect a large and significant break in the interest rate level effect on our indicators of lower bound constrained interest rates. Distributions of low interest rates after this break no longer show substantial signs of compression at a lower bound. We estimate this structural change to have happened in 2014/2015 over the exact same period when interest rates in the Eurozone and other major economies started to violate the zero lower bound. These results indicate that investors both in the US and the Eurozone were forced to revise their formerly held zero lower bound beliefs in light of opposing evidence.

Our results appear not to be driven by time-variation in risk premia. Firstly, we theoretically show that we can use option-implied skewness and kurtosis of the risk-neutral distribution $\mathbb{Q}$ to infer an upper limit for the lower bound on interest rates that is equally valid for the physical distribution $\mathbb{P}$. Secondly, we also analyze the characteristics of physical interest-rate distributions. To this end, we estimate realized skewness using the approach of Neuberger (2012) and realized kurtosis using the approach of Bae and Lee (2021). Overall, the evaluation of physical distributions leads us to conclusions very similar to those we draw from risk-neutral distributions.

Understanding the impact of a lower bound on interest-rate distributions is important for analyzing and modeling the dynamics of the yield curve and therefore has implications for a broad range of related questions. Most generally, the estimation of dynamic term-structure models critically depends on reasonably accounting for a dynamic lower bound effect. Dynamic term-structure models are for example used by central banks to extract market expectations of future interest rates and inflation, to estimate risk premia, and to evaluate the success of their monetary policy. Furthermore, corporations rely on term-structure models to manage their interest-rate risk. Misspecification of the lower bound induced interest-rate asymmetry may materially distort risk-management decisions.

More particularly, shadow-rate models have been employed to generate alternative indicators summarizing monetary activities when the short rate is constrained by the lower bound. Most prominently, the shadow-short rate (SSR) is frequently used as generated regressor in macroeconomic studies to capture the effect of unconventional monetary policy. This practice relies on the assumption of a lower-bound constrained short rate and empirical results have been shown to be extremely sensitive to the lower bound specification (see Krippner (2015, 2017)). Moreover, the strength of a lower bound impact likely has important implications for the dynamics of risk premia embedded in interest rates. Within a shadow-rate framework for example, a tightly binding lower bound leads to the compression of risk premia and thereby induces a dependence between the level of interest rates and excess returns. Assessing to what degree current interest rates reflect market-participants' lower bound considerations also has direct practical consequences for the extraction of likely short-rate paths from futures prices: the distributional asymmetry at a lower bound drives a wedge between the expected value of future yields and their most likely realization.

Beyond interpretation and modeling of interest-rate *levels*, accounting for a lower bound impact is also crucial to understand interest-rate *volatility*. The large decline in interest-rate volatility during the period of low interest rates poses the question whether we experienced a corresponding decrease in underlying macroeconomic uncertainty or whether we simply observed compression at a lower bound. Distinguishing these effects is important when analyzing the relation between economic growth and uncertainty, where interest-rate volatility provides a natural candidate for measuring uncertainty (see e.g. Creal and Wu (2017) and Istrefi and Mouabbi (2018)). Additionally, the understanding of lower bound effects contributes to the literature on unspanned stochastic interest-rate volatility. Proximity to the lower bound compresses volatility and can be expected to decrease sensitivity to unspanned factors. Therefore, the portion of volatility variation spanned by the term structure should be increasing with interest rates approaching their lower bound. In fact, Filipović et al. (2017) document a substantial increase in volatility-level dependence during the near-zero interest-rate period in the US and attribute this phenomenon to the zero lower bound.

Related Literature Our paper contributes to the large body of literature that studies the role of a lower bound on nominal interest rates. A vast macro literature with seminal contributions by Krugman, Dominquez, and Rogoff (1998) and Eggertsson and Woodford (2003) has emphasized the economic importance of a lower bound. Consequently, a wide range of empirical facts during the last decades of ultra-low interest rates has been explained by the presence of a (zero) lower bound. Bauer and Rudebusch (2016) estimate a large impact on monetary policy expectations due to near-zero interest rates. King (2019) finds that the zero lower bound significantly weakened the duration channel of bond supply. Using counterfactual experiments, Gust et al. (2017) estimate that the zero lower bound constraint was responsible for about 30% of the US GDP contraction during the Great Recession. Chung, Laforte, Reifschneider, and Williams (2012) conclude that the ZLB had a first-order impact on macroeconomic outcomes in the United States. Christiano, Eichenbaum, and Rebelo (2011) measure a substantially increased government-spending multiplier near the zero lower bound. Analyzing high frequency data, Swanson and Williams (2014) show that long-term yields became less sensitive to macroeconomic announcements after 2008. Datta, Johannsen, Kwon, and Vigfusson (2021) argue that the zero lower bound induced stronger positive correlation between oil and equity returns and increased their sensitivity to macroeconomic news. Gourio and Ngo (2020) establish that the zero lower bound environment created positive correlation between inflation and stock returns. All of these studies crucially rely on assumptions about the location of the lower bound and its incorporation by markets participants. We aim to provide missing empirical evidence for these (zero) lower bound beliefs.

Numerous studies explore the impact of a lower bound through lower bound consistent term-structure models fitted to the yield curve. These papers essentially rely on assumptions about the location of the lower bound to infer the model-implied impact on the yield-curve. For the US, Japan, and the UK, the lower bound is commonly assumed to be zero or slightly above. Examples include Krippner (2012, 2013); Christensen and Rudebusch (2015); Bauer and Rudebusch (2016); Wu and Xia (2016) for the US; Ichiue, Ueno, et al. (2013); Kim and Singleton (2012) for Japan; and Andreasen and Meldrum (2015); Carriero, Mouabbi, and Vangelista (2018) for the UK. Some studies treat the lower bound as a free model-parameter (see Christensen and Rudebusch (2016); Kim and Priebsch (2013); the online implementation of Wu and Xia (2016) for the euro area), hence estimate its location from yield-curve data. None of these term-structure model applications, however, provides direct evidence for their respective lower bound assumptions. We fill this gap as we gather direct evidence from option-implied distributions without relying on model assumptions.

Evidence from option markets regarding a lower bound on interest rates has been studied before by Filipović et al. (2017), Mertens and Williams (2021), and Bauer and Chernov (2022). Our paper contributes to this literature in several ways. Most importantly, our paper explicitly addresses the violation of the zero lower bound and examines the implications for option-implied distributions. We provide first-time evidence for a change in lower bound beliefs related to this zero lower bound violation. For US markets, considering a departure from the zero lower bound conviction sets our paper apart from prior literature. Moreover, our study extends existing option-based evidence for the US market to the European market, allowing us to analyze lower bound effects in a negative interest rate environment. Regarding the empirical methodology, Mertens and Williams (2021) and Bauer and Chernov (2022) also use skewness as indication for lower bound constrained yields. Their use of skewness is motivated by an interest rate model with a simple truncation mechanism, generating asymmetric distributions at the lower bound. We theoretically prove that these asymmetric distributions are not just the result of simplistic modelling assumptions but are - to a certain, quantifiable extent - a model-independent feature of lower bound constrained yields. Beyond focusing on the distribution asymmetry (skewness) we also consider the conditional level-volatility dependence as in Filipović et al. (2017) and additionally the fat tails (kurtosis). This provides us with a rich characterization of lower bound constrained distributions and hence helps us to identify lower bound effects more sharply than previous literature. Our paper also differs from the literature in the estimation of interest-rate distribution moments. Filipović et al. (2017) use model-implied ATM volatilities and Mertens and Williams (2021) calculate the moments from a full probability density function fitted to option prices. In contrast, we directly estimate model-free moments from option prices which has shown to deliver robust results and hence became the standard approach in the literature. Bauer and Chernov (2022) use a similar model-free methodology but apply it to options on treasury futures instead of options on short-term interest rates as we do. For our purposes, options on short-term rates come with two advantages: (1) they capture lower bound effects more precisely, since short-term rates are more directly affected by a lower bound than long-term rates. (2) We do not need an approximate mapping from the futures price distribution to an interest-rate distribution as is required for options on treasury futures.

Few studies have addressed the violation of the zero lower bound. Lemke and Vladu (2017), Kortela (2016), Wu and Xia (2017), and Geiger and Schupp (2018) estimate lower-bound consistent term-structure models for the euro area. They all assume a time-varying lower bound to accommodate for the fact that euro area interest rates incrementally entered negative territory. A central input in these studies are estimates for the time-varying location of the lower-bound for which they rely on a variety of proxies. Our findings are consistent with the assumption of a time-variation of the lower-bound. However, contrary to assumptions in this literature, our results suggest that with the violation of the zero lower bound, markets revised their beliefs to such an extent that any lower-bound effects became negligible afterwards. In other markets, the violation of the zero lower bound has been largely ignored so far. Even after the general validity of the zero lower bound assumption has been proven wrong in the Eurozone, numerous studies still rely on its unchanged validity in other markets where this bound has not been breached yet. Examples include Caballero and Simsek (2021); Johannsen and Mertens (2021); Andreasen and Meldrum (2019); Mertens and Williams (2021) for the US and Iiboshi, Shintani, and Ueda (2022) for Japan. We find this assumption to be inconsistent with evidence from option markets which suggests a revision of lower-bound beliefs not only in the Eurozone but also in the US where interest rates remained positive until now.

Regarding our model-free methodology to estimate option-implied interest-rate moments our paper builds on Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003). Recent studies applying similar methods to interest-rate distributions include Trolle and Schwartz (2014), Choi, Mueller, and Vedolin (2017), Bauer, Lakdawala, and Mueller (2022), and Bauer and Chernov (2022).

2 Interest-rate distributions at the lower bound

In this section we characterize interest-rate distributions close to the lower bound in terms of empirically observable measures. This provides the foundation for our empirical analysis where we compare option-implied distributions to these theoretical characteristics to make inferences about markets lower bound beliefs. Our characterization relies on a model-free description of the interest-rate distribution *shape* and established implications for the dynamic relationship between the expectation and volatility of interest-rate distributions.

2.1 Shape of interest-rate distributions close to the lower bound

We exploit the fact that close to the lower bound interest-rate distributions exhibit a very distinctive shape. They are highly asymmetric and fat tailed. Importantly, this is not just the prediction of a certain model, but a direct and general implication that follows immediately from the definition of a lower bound. We start by developing some intuition of this theoretical result and then present formal proof.

Market participants anticipate a lower bound on interest rates by excluding the possibility of future interest-rate realizations below such a floor in their decision making. When market expectations of interest rates are well above their floor relative to the associated uncertainty, the impact on the shape of the interest-rate distribution is likely to be negligible. However, as interest rates approach their lower bound, the range of realizations around the expectation conceived possible by the market becomes increasingly constrained. When interest-rate expectations are close to the lower bound, a high level of uncertainty may only come from the probability of large upward deviations from the mean, as downward deviations are limited. With interest rate uncertainty necessarily driven mostly by extreme positive deviations at the lower bound, the distribution shapes are by definition asymmetric and fat tailed. This intuition holds for any distribution, whether it incorporates risk premia or not (i.e., for $\mathbb{P}$ and $\mathbb{Q}$ distributions alike). The shape characteristics of interest rates close to the lower bound can be concisely summarized in terms of the corresponding central moments: interest rates constrained at a lower bound exhibit high positive skewness and high (excess) kurtosis. This insight has immediate empirical implications: When market participants incorporate a lower bound in their investment decisions, this is directly reflected by the prices of interest rate contingent claims. Therefore, at the market anticipated lower bound, option-implied interest-rate distributions are characterized by high positive skewness and high kurtosis.

In the following we formalize our intuition about the relationship between proximity to the lower bound and the shape characteristics of interest-rate distributions. Our derivations are completely model-free, i.e. we make no assumptions about the interest-rate distribution other than the existence of its finite mean and variance. The distribution may be any continuous, discrete, or mixed distribution. Hence, we also do not need to rely on any a-priori assumption on the mechanism through which a lower bound might shape interest-rate distributions. In particular, note that our derivations equally apply to interest-rate distributions under the physical measure $\mathbb{P}$ as well as to option-implied distributions under the risk-neutral measure $\mathbb{Q}$. We will show that interest-rate distributions close to the lower bound are always asymmetric and that their central moments skewness and kurtosis are bounded from below. We are able to derive explicit expressions for these bounds and specify them in terms of the distance of interest-rate expectations to the lower bound relative to their volatility.

Consider L_T the LIBOR rate fixed at T for the period T,T+Δ with conditional date-t probability distribution $P_t \in \mathcal{P}$. $\mathcal{P}$ is the set of all probability measures with support on $[LB, \infty)$ and with finite and given mean μ and variance σ^2. Otherwise, the distribution P_t is allowed to be completely arbitrary. LB is the respective LIBOR-rate floor and $d = \frac{(\mu - LB)}{\sigma}, d \in (0, \infty)$ denotes the distance of the distribution mean to the floor in terms of its standard-deviations.
We start by noting that

PROPOSITION 1. *For $0 < d < 1$, the LIBOR-rate distribution P_t cannot be symmetric*[1].

Proof. See Appendix A.1

Next, we derive explicit lower bounds for skewness and kurtosis of the LIBOR-rate distribution. Our main result is:

PROPOSITION 2. *Let $\underline{S}$ and $\underline{K}$ denote lower bounds for skewness and kurtosis respectively over the set $\mathcal{P}$, i.e.*

[1] P_t is said to be symmetric if and only if $\forall \delta \in \mathbb{R} : f(\mu - \delta) = f(\mu + \delta)$ where f is the probability density function or the probability mass function respectively.

$$\underline{\mathcal{S}} = \inf_{P \in \mathcal{P}} \mathbb{E}^P \left[\left(\frac{L_T - \mu}{\sigma} \right)^3 \right] \tag{1}$$

$$\underline{\mathcal{K}} = \inf_{P \in \mathcal{P}} \mathbb{E}^P \left[\left(\frac{L_T - \mu}{\sigma} \right)^4 \right]. \tag{2}$$

Then the solution to (1),(2) *exists and is given by*

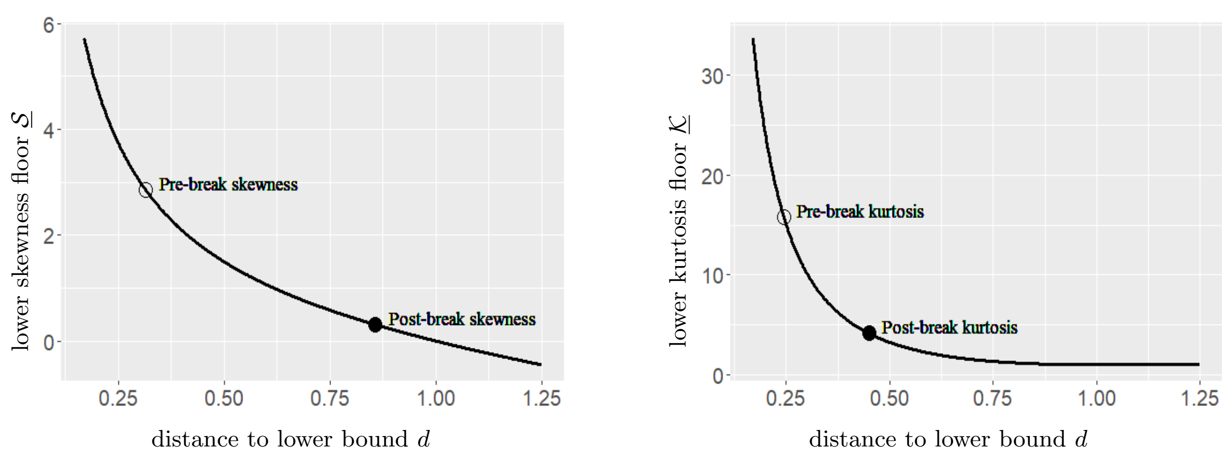
$$\underline{\mathcal{S}} = \frac{1 - d^2}{d},$$

$$\underline{\mathcal{K}} = \begin{cases} 1 & \text{for } d \geq 1 \\ \frac{1+d^6}{d^4+d^2} & \text{for } 0 < d < 1. \end{cases}$$

Proof. See Appendix A.2

Figure 1 displays derived central moment bounds for a varying distance of the distribution-mean to the lower bound. Note that the bounds $\underline{\mathcal{S}}$ and $\underline{\mathcal{K}}$ are only feasible for rather pathological interest-rate distributions where all the mass is placed on only two points. Restricting interest-rate distributions to more realistic shapes would of course further tighten the central-moment-bounds at the interest-rate floor.

Figure 1: Central-moment-floors near the lower bound



This figure shows the theoretical floors of skewness $\underline{\mathcal{S}}$ (left panel) and kurtosis $\underline{\mathcal{K}}$ (right panel), depending on the distance d of the distribution mean to the lower bound in terms of standard deviations.

Derived relationships enable us to empirically identify interest rates close to their lower bound simply based on the central moments of their distribution. Interest-rate distributions tightly constrained by a lower bound necessarily exhibit both high positive skewness and kurtosis. At any point in time, we can therefore assess whether empirically observed (implied) distributions are theoretically consistent with markets anticipating a lower bound in close proximity to current expectations.

For (implied) interest-rate distributions not exhibiting high levels of skewness and kurtosis we can exclude the possibility of a tightly binding lower bound. Furthermore, we can quantify the minimal remaining distance of expectations to the lower-bound, respectively, the model-free maximum lower-bound location. We will use these theoretical relationships to evaluate how lower-bound beliefs may have developed over time.

Our theoretical analysis of distribution shapes is useful to empirically distinguish in a model-agnostic way between interest rates stuck right at their lower bound - where skewness and kurtosis become explosive - and interest rates with still some space left to move downwards. At interest-rate levels more moderately away from the lower bound, however, the model-free impact on distributions becomes too weak as to expect strong further guidance from skewness and kurtosis regarding the exact distance to the lower bound.

2.2 Level-volatility dependence

Contrary to skewness and kurtosis, there is no similar model-free bound for the volatility of distributions close to the lower bound. Highly skewed distribution shapes could theoretically still produce arbitrarily high volatility even when expectations are very close to the lower bound. However, several studies have argued for a strong positive level-volatility dependence as a feature of lower-bound constrained yields (Kim and Singleton (2012), Filipović et al. (2017), King (2019), Kim and Priebsch (2013), Andreasen and Meldrum (2019), Christensen and Rudebusch (2015)). Intuitively, as interest-rate expectations increase, distributions become less constrained giving ceteris paribus rise to a higher volatility. And even though the level-volatility dependence of interest rates is not a fully model-independent feature of yields near the lower bound, it also does not depend on the exact way the lower bound is enforced in an interest-rate model. In fact, high level-volatility dependence close to the lower-bound has been shown to emerge from every popular lower-bound consistent dynamic term-structure model.[2] We will therefore consider the conditional level-volatility dependence of interest rates in addition to the derived model-free shape characterizations to identify lower-bound constrained yields.

3 Option-implied lower bound beliefs

3.1 Data

Our analysis uses futures and options on short-term interest rates for the US and the Eurozone market. We obtain daily settlement prices of futures on the three-month USD-LIBOR rate (Eu-

[2]For a discussion of volatility compression at the lower bound see e.g. Kim and Priebsch (2013), Andreasen and Meldrum (2019), Kim and Singleton (2012) and Christensen and Rudebusch (2015) for shadow rate models; Kim and Singleton (2012) for affine term structure models with square-root processes as in Cox, Ingersoll, and Ross (1985) and Dai and Singleton (2000); Andreasen and Meldrum (2019) and Kim and Singleton (2012) for quadratic term structure models as in Ahn, Dittmar, and Gallant (2002) and Leippold and Wu (2002); Filipović et al. (2017) for linear-rational models; Monfort, Pegoraro, Renne, and Roussellet (2017) for their Autoregressive Gamma-zero (ARG_0) framework; and Feunou, Fontaine, Le, and Lundblad (2022) for their tractable term structure framework.

rodollar futures) and the three-month Euribor rate, as well as options written on those futures. Data comes from Thomson Reuters Datastream and covers the period from June 2006 (May 2005) to December 2019 for the Eurodollar (Euribor) contracts. Eurodollar contracts traded at the CME and Euribor contracts traded at ICE Futures Europe constitute two of the largest markets for short-term interest rates in the world. Table A.1 in the Appendix reports trading statistics for December 2019 according to CME and ICE Futures Europe. To set these numbers into perspective we also list figures for US Treasury futures and options. Trading volumes of Eurodollar futures and options even exceeds those of the highly liquid Treasury note contracts. Naturally, trading in Euribor contracts is a bit more muted but average daily volume in these futures (options) still amounted to 537,316 (49,195) trades. These figures indicate that liquidity in LIBOR futures and options compares quite well with other segments of the interest rate market which ensures reliable market prices in our sample.

Payoffs are directly tied to the underlying reference rate (L_t) as futures contracts settle in cash based on 100–L_T. Futures prices are quoted as $100 - f_{t,T}$, where $f_{t,T}$ is the date-t future LIBOR rate with maturity T. In the remainder we will directly refer to $f_{t,T}$ as the futures rate. The underlying reference rate for the futures and option contracts is the three-month LIBOR (Euribor) rate, a daily benchmark rate representing unsecured interbank lending conditions in the US (Eurozone). It is the key cash-market benchmark within USD (EUR) money markets, serving as reference rate for forward rate agreements, interest rates swaps, futures, and options. Hence our futures and options sample directly captures market expectations of the most important short-term interest rate in these two economies.

For each trading day we obtain estimates of the option-implied moments $Vol_{t,T}(L_T)$, $Skew_{t,T}(L_T)$ and $Kurt_{t,T}(L_T)$ for all available quarterly contract expirations. We limit our analysis to quarterly expiries as these contracts are most liquid and allow for straightforward estimation of implied moments. Quarterly Eurodollar (Euribor) options have maturities of up to four (two) years, providing us with 16 (8) points across the short-end of the yield curve at any given day. Appendix A.3 gives further details on our data selection procedure and the composition of our sample.

We estimate option-implied moments with a model-free methodology using the insights of Bakshi and Madan (2000), Carr and Madan (2001), Bakshi et al. (2003) and Jiang and Tian (2005). An application of their fundamental result allows us to derive formulas for volatility, skewness and kurtosis of the risk-neutral LIBOR rate distribution at given horizon T in terms of the corresponding futures rate and the cross-section of options with the same maturity written on those futures. Details of our model-free methodology are provided in Appendix A.4.

3.2 Time variation in implied skewness, kurtosis and the level-volatility relation

Figure 2 plots time-series of the interest futures rate in the US and the Eurozone, together with the three measures that we use to assess ex-ante beliefs about the lower bound: skewness, kurtosis and the level-volatility relationship. For illustration purposes we present time-series of option-implied skewness, kurtosis and the interest rate futures rate that refer to a constant maturity of one year.

These constant maturity values are obtained by linearly interpolating between available adjacent maturities. To measure the level-volatility relationship, we follow Filipović et al. (2017) and regress changes in volatility on changes in the futures rate,

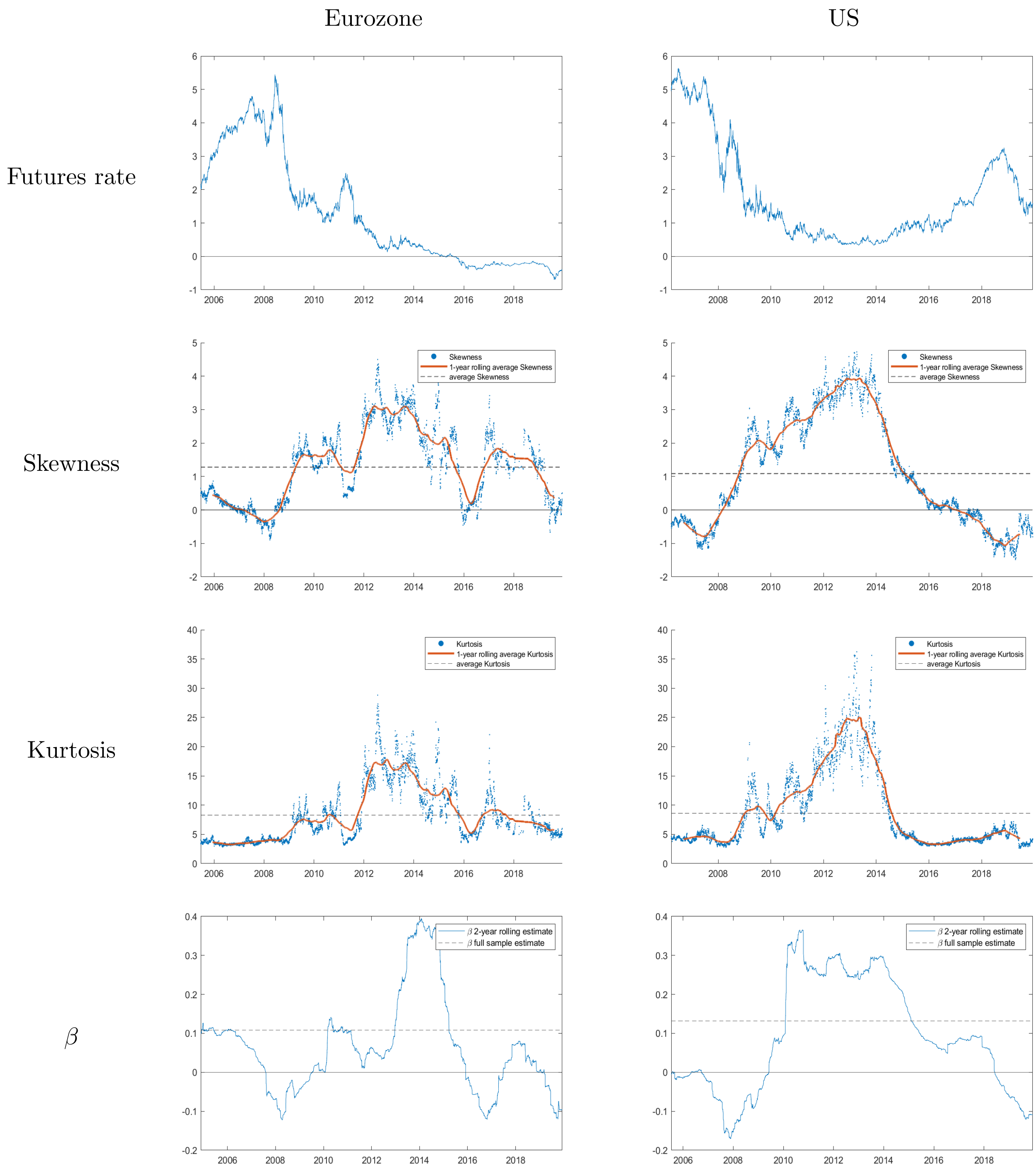
$$\Delta Vol_{i,t} = \alpha + \beta \Delta f_{i,t} + \epsilon_{i,t} \tag{3}$$

In the last row of Figure 2 we show 2-year rolling estimates of β.

Unconditionally, implied skewness and excess-kurtosis are slightly positive, confirming the results of Trolle and Schwartz (2014) for the swaption market. The level-volatility regression coefficient β is somewhat positive in the unconditional estimation. However, all three measures show substantial time-variation over our sample. Very notably, all three indicators are highly elevated during the period of near-zero interest rates - in the US 2008 – 2015 and in the Eurozone 2012 – 2015. Consistent with a tightly binding zero lower bound constraint, skewness, kurtosis and β all hover around 3-4 times their average values during this period. In contrast, prior to the respective zero-bound periods, skewness is close to zero or even negative in both markets, excess-kurtosis is still positive but much more muted and the level dependence of volatility fluctuates around zero. This combined evidence clearly speaks in favor of anticipation of a zero lower bound resulting in tightly constrained distributions for interest rates close to zero. However, all measures experience a sharp drop around 2014/2015 down to levels similar to those observed prior to the zero lower bound period. Strikingly, this drop coincides with the ECB subsequently lowering Eurozone short-term rates into negative territory, hence breaching the zero lower bound. This simultaneous decline in all indications of a constraining lower bound as Eurozone interest rates continue to fall has revealing implications for markets lower bound beliefs: It is inconsistent with the idea that markets always anticipated a somewhat negative lower bound as this would result in more – not less – constrained distributions as interest rates become negative. It is also inconsistent with the idea that markets made only small subsequent readjustments to their zero lower-bound beliefs and repeatedly had to adapt convictions as they were again and again "surprised" by the experience of ever more negative interest rates. Instead, little indication for lower bound constrained interest rates after 2015 suggests that markets did not continue to anticipate a lower-bound close to observed interest rates. A more plausible explanation seems to be that market participants substantially revised their lower-bound beliefs as the previously held zero-lower bound conviction was contradicted.

In the US, the rapid decline of skewness, kurtosis and the level-volatility dependence *precedes* the lift-off of the Fed policy rate by more than a year. With US interest rates still about as close to zero as during the 2008 – 2015 period until the end of 2016, all lower-bound indicators fall to substantially below-average levels already by the end of 2014 when interest rates in the Eurozone became negative. This means that the disappearance of lower-bound constrained distributions in the US was not merely a result of increasing interest rates. It seems that the breach of the zero lower bound in the Eurozone also prompted US investors to materially revise their convictions. This interpretation is also supported by the developments towards the end of our sample: When

Figure 2: Time-series of lower bound indicators



This figure shows time-series of skewness, kurtosis and β together with the level of the futures interest rate for both the US and the Eurozone market. The futures rate, skewness and kurtosis are each interpolated between adjacent maturities to a constant maturity of one year. Red lines show 252-day moving average values. The last row displays β from a rolling 2-year estimation of regression (3).

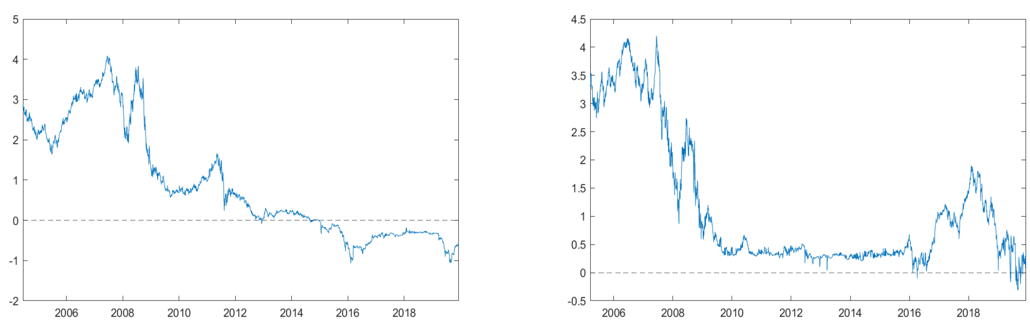
US interest rates in 2019 returned to levels as low as in the end of 2008, skewness, kurtosis and β remained at about their lowest levels in our sample, with skewness and β being even negative.

3.3 Option-implied limits for lower bound beliefs

Leveraging derived floors $\underline{\mathcal{S}}$ and $\underline{\mathcal{K}}$, Figure 3 shows the option-implied *maximum* lower-bound that would be consistent with observed skewness and kurtosis at any point in time. During the period 2008-2015, observed distributions are consistent with a lower-bound at or even slightly above zero. This provides model-free confirmation that the zero lower bound very well might have been incorporated by markets at that time. Beginning in 2015, the maximum lower-bound in the Eurozone drops into deeply negative territory, reaching less than -1% in early 2016. This implies that we can model-independently exclude the possibility of an only somewhat negative lower bound close to experienced interest rate levels in the Eurozone: After the breach of the zero lower bound, markets quickly adjusted their lower-bound beliefs to *at least* -1%. In the US, the maximum lower bound also drops to slightly negative levels in 2016, and more clearly at the end of 2019, reaching as low as -0.3%. The continued anticipation of a zero lower bound in the US can therefore be ruled out.

Figure 3: Option-implied maximum lower bound

Eurozone | US

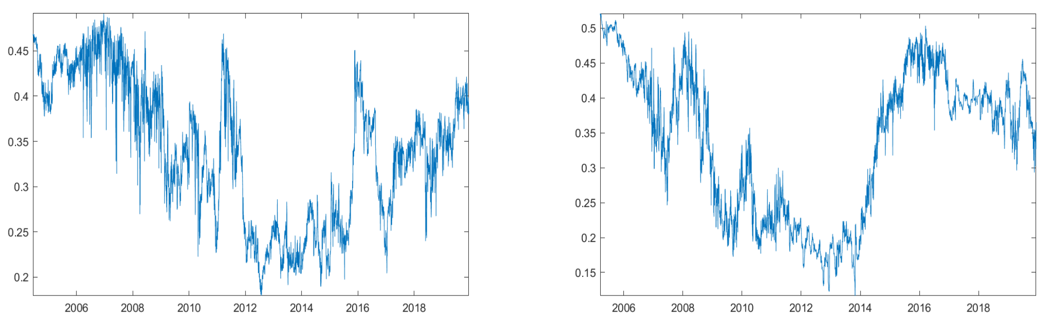


This figure shows the maximum possible market anticipated interest rate lower bound implied by option skewness and kurtosis. At each point in time, we calculate for each maturity the highest value of a lower bound which would still be consistent with option-implied moments. To do this, we invert relationships $\underline{\mathcal{S}}$ and $\underline{\mathcal{K}}$ to extract the minimal distance to the lower bound d from option-implied skewness and kurtosis. Together with the futures rate and option-implied volatility we can then directly recover the maximum lower bound consistent with skewness respectively kurtosis for any given maturity on a given date. The lowest value for the maximum lower bound across all maturities on a given date gives us the sharpest upper limit for the lower bound on that day.

Another way to look at the data is to consider the minimal distance d of distribution means to the lower bound in terms of standard deviations. This implied minimal distance is plotted in Figure 4 for the potentially most constrained maturity at each point in time. This measure highlights the potential importance of the zero lower bound during the 2008-2015 period where distributions are consistent with highly constrained interest rates of less than 0.2 standard deviations above the lower bound. After 2015, the importance of a lower bound quickly decreases as the minimal distance reverts to pre-2008 levels. Hence, distributions after 2015 imply a much less constraining lower bound with even the potentially most constrained maturities having at least 0.4 standard deviations room to move downward.

Figure 4: minimal distance to lower bound

Eurozone US



This figure shows the minimal possible distance d in terms of standard deviations between the futures rate and the lower bound implied by option skewness and kurtosis. At each point in time, we calculate for each maturity the smallest distance d which would still be consistent with option-implied moments. To do this, we invert relationships $\underline{S}$ and $\underline{K}$ to extract the minimal distance to the lower bound d from option-implied skewness and kurtosis. This directly gives us the minimal distance d consistent with skewness respectively kurtosis for any given maturity on a given date. On a given date, we plot the lowest value for the minimal distance d across all maturities, giving us the minimal possible distance d of the potentially most constrained maturity on that day.

3.4 Distribution characteristics and the level of interest rates

To support our graphical evidence more formally, we now look at how implied distribution characteristics depend on the level of interest rates. Table 1 and Table 2 show sample means of risk-neutral skewness and kurtosis conditional on the level of the futures rate across all maturities. They also report Newey-West t-statistics of the regressions $M_{t,T} = \alpha + \beta I_{\left(a \leq f_{t,T} < b\right)}$, testing the difference in means between the conditioning group and the remainder of the sample.

While unconditionally slightly positive, implied skewness substantially varies with the level of the interest rate. Conditional on interest rates being far above any potential lower bound, skewness in both markets is negative on average. This suggests that apart from the impact of a lower bound, market participants on average are willing to pay more for OTM options betting on falling interest rates than for equivalent OTM options betting on rising interest rates. Intuitively, this negative implied interest-rate skewness matches with the experience that sharp short-rate cuts

Table 1: Conditional skewness

	Total	<0%	0%-2%	2%-4%	>4%
Panel A: US					
Mean	0.94	-	1.65	0.14	-0.49
t-stat	-	-	9.29	-6.53	-9.78
stdv	1.47	-	1.48	0.79	0.44
N	37646	0	21472	12414	3760
Panel B: EU					
Mean	1.22	0.99	2.04	0.51	-0.12
t-stat	-	-1.4	10.05	-6.53	-12.49
stdv	1.09	0.82	0.86	0.52	0.39
N	25768	5665	11413	5781	2909

This table shows summary statistics of option-implied skewness for the US and the Eurozone across all available maturities. The first column displays unconditional results while the second to fifth column display results conditional on the futures rate being below 0%, in the interval 0%-2%, in the interval 2%-4% and above 4%, respectively. The second row within each panel shows t-statistics of the slope coefficient from the regression $Skew_{t,T} = \alpha + \beta I_{(a \leq f_{t,T} < b)}$. t-statistics are corrected for heteroscedasticity and serial correlation up to 252 lags (i.e. one year) using the approach of Newey and West (1987).

are much more likely than equivalent sharp increases.[3] However, with interest rates approaching zero, skewness increases and turns positive. Conditional on interest rates being in the interval 0%-2%, skewness is clearly positive and substantially higher than average in both markets. Also, the difference in conditional means is highly significant in both cases. This is in line with our previous observation of a tightly constraining zero-lower-bound constraint for most of the near-zero interest rate period. Average skewness for negative interest rates in the Eurozone, however, reveals a discontinuity in this negative level-skewness relationship. Conditional on interest rates being below 0% in the Eurozone, average skewness more than halves compared to its value for positive near-zero interest rates. Hence, static level-dependence induced by anticipation of a fixed lower-bound is unable to satisfyingly explain observed empirical patterns in interest rate skewness. Accounting for a potential shift in lower-bound views requires to allow for a change in the interest-rate level effect of skewness over time. We will consider this approach in the next section.

Level-conditional results for implied kurtosis show a very similar pattern. Kurtosis is on average as well as conditional on rates being safely away from a lower bound in excess of three. This indicates interest-rate distributions with heavier tails than the normal distribution independently from a lower-bound explanation. Again this finding is in line with the results of Trolle and Schwartz (2014). With falling interest rates, excess kurtosis becomes more pronounced. Conditional on interest rates being close to but above zero, kurtosis is significantly above average in both markets, consistent with (zero-)lower-bound-induced heavier right tails. As for skewness, average kurtosis for negative interest rates contradicts with a fixed lower bound explanation: For negative interest

[3] Note that for this intuition of negative interest-rate skewness – namely negatively skewed short-rate increments – the effect should be expected to decrease with maturity. This is what we observe in Table 4 and 5

Table 2: Conditional kurtosis

	Total	<0%	0%-2%	2%-4%	>4%
Panel A: US					
Mean	7.61	-	10	4.49	4.27
t-stat	-	-	7.31	-6.78	-6.39
stdv	6.61	-	7.9	1.12	1.16
N	37646	0	21472	12414	3760
Panel B: EU					
Mean	7.73	7	10.84	4.19	4
t-stat	-	-1.29	8.47	-8.87	-8.54
stdv	4.53	2.65	4.72	0.97	0.88
N	25768	5665	11413	5781	2909

This table shows summary statistics of option-implied kurtosis for the US and the Eurozone across all available maturities. The first column displays unconditional results while the second to fifth column display results conditional on the futures rate being below 0%, in the interval 0%-2%, in the interval 2%-4% and above 4%, respectively. The second row within each panel shows t-statistics of the slope coefficient from the regression $Kurtosis_{t,T} = \alpha + \beta I_{(a \leq f_{t,T} < b)}$. t-statistics are corrected for heteroscedasticity and serial correlation up to 252 lags (i.e. one year) using the approach of Newey and West (1987).

rates, the sample mean of kurtosis is below average and substantially lower than for positive near-zero rates, indicating less heavy distribution tails despite lower interest rates. Hence our estimates for conditional kurtosis are also inconsistent with an increasingly approached negative lower bound but point to a potential shift in lower-bound perceptions.

Table 3 contains our results for the level-conditional version of regression (3). Unconditionally, as well as far distant from a potential lower bound, volatility shows little relation with interest rate levels. Unconditional coefficients are significantly positive in both markets but small in magnitude and R^2's are low. This is consistent with the large literature on unspanned stochastic interest-rate volatility which documents that the bulk of volatility variation in yields cannot be explained by variation in the term structure (see e.g. Collin-Dufresne and Goldstein (2002), Heidari and Wu (2003), Li and Zhao (2006, 2009)). Now, as we condition on the futures rate being in the interval 0%-2%, coefficients strongly increase and are highly significant, and R^2's are substantially higher. This replicates the finding of Filipović et al. (2017) and might be evidence for a zero-lower-bound-induced level-volatility dependence. Conditional on negative interest rates in the Eurozone, however, coefficients are (again) small in magnitude, statistically not significant and R^2's are close to zero. In line with evidence for skewness and kurtosis, any potential lower-bound effect on level-volatility dependence (for positive near-zero rates) apparently did not carry into a negative interest-rate environment.

In summary, all three empirical measures indicate a substantial lower-bound impact on interest-rate distributions close to zero. However, level-conditional distributions also reveal a strong decrease of lower-bound effects for negative interest rates. This result cannot be explained by market an-

Table 3: Conditional level-dependence of volatility

	Total	<0%	0%-2%	2%-4%	>4%
Panel A: US					
α	-0.001***	-	-0.001***	0	0
	(-3.124)	-	(-3.664)	(0.485)	(0.037)
β	0.132***	-	0.262***	0.088***	-0.129***
	(12.819)	-	(23.669)	(5.445)	(-6.253)
R^2	0.075	-	0.235	0.042	0.08
N	37580	0	21512	12327	3741
Panel B: EU					
α	0**	-0.001***	0	0	0.001
	(-2.468)	(-3.142)	(-1.64)	(-0.799)	(0.857)
β	0.109***	-0.05	0.282***	0.038**	0.008
	(8.738)	(-1.473)	(14.973)	(2.492)	(0.281)
R^2	0.033	0.005	0.154	0.006	0
N	25711	5770	11290	5764	2887

This table shows the results from regressing daily changes in volatility on changes in the futures rate. The first column displays unconditional results while the second to fifth column display results conditional on the futures rate being below 0%, in the interval 0%-2%, in the interval 2%-4% and above 4%, respectively. t-statistics of the coefficients are shown in parentheses and are corrected for heteroscedasticity and serial correlation up to two lags using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively.

ticipation of a static lower bound. Instead, graphical investigation of the time-series of skewness, kurtosis and β suggests a change of the market's lower bound anticipation over time. In the next section we will therefore formally investigate whether a shift in lower-bound views can explain the observed patterns in skewness, kurtosis, and level-volatility dependence.

4 Structural break

The previous analysis shows that market anticipation of a fixed lower bound is unable to fully explain observed implied distribution characteristics, particularly for periods of negative interest rates in the Eurozone. In this section we therefore formally analyze whether market participants revised their lower bound beliefs during the sample period. To account for a potential shift of a lower bound over time we extend the level-conditional analysis of the previous section towards a structural-break regression framework. Our goal is to analyze whether and when the observed lower-bound impact on distributions of low interest rates has been subject to a change over time. To this end, we consider the following model specification:

$$M_{t,T} = \alpha + \beta_0 I_{\left(f_{t,T}<\delta\right)} I_{(t<T_b)} + \beta_1 I_{\left(f_{t,T}<\delta\right)} I_{(t\geq T_b)} + \epsilon_{t,T} \tag{4}$$

$M_{t,T}$ denotes the distribution moment of interest in t for option maturity T and $I_{\left(f_{t,T}<\delta\right)}$ is an indicator variable identifying corresponding interest rates potentially close to the lower bound. α, which we assume to be constant over the sample, is the mean of $M_{t,T}$ without the impact of a lower bound. β_0 and β_1 estimate the impact of the lower bound on $M_{t,T}$ before and after the break respectively. Hence $\beta_1 - \beta_0$ quantifies magnitude and direction of the break. Regarding the timing of the break T_b we remain completely agnostic and estimate it endogenously from the data. For each pooled regression (4) we estimate T_b as a common breakpoint to all individual maturities using the least-squares estimator of Bai and Perron (1998). The extension of this breakdate estimator to the panel data case relevant to our application was studied in Bai (2010) and Baltagi, Feng, and Kao (2016). The OLS estimator of T_b is given by $\hat{T}_b \; = argmin_{1<T_b\,<\,T-1} SSR\,(T_b)$, where $SSR\,(T_b) = \sum_{t=1}^{T}\sum_{i=1}^{N}\left(M_{t,T} - \hat{\alpha} - \widehat{\beta}_0 I_{\left(f_{t,T}<\delta\right)} I_{(t<T_b)} - \widehat{\beta}_1 I_{\left(f_{t,T}<\delta\right)} I_{(t\geq T_b)}\right)$. To assess significance of the estimated breaks we consider the SupF-test proposed by Andrews (1993), Andrews and Ploberger (1994) and Bai and Perron (1998). For each univariate time-series of individual maturity T we run an Wold-test under the null of no structural change against the alternative hypothesis of a break at the date estimated from the pooled regression (4). Critical values for the SupF-test are reported in Bai and Perron (1998).

Our specification requires the choice of a cut-off level δ to distinguish between low interest rates potentially subject to substantial lower-bound effects and interest rates far above the lower bound where such an impact is negligible. This accounts for the explosive behavior of the derived lower-bound constraints $\underline{S}$ and $\underline{K}$ which only become relevant very close to the lower bound. In our main analysis we set $\delta = 2\%$. On the one hand, this provides us with sufficient observations for reliable estimation. On the other hand, our choice should achieve satisfying sharp identification as almost all observations for interest rates below 2% fall into periods when the corresponding central bank rate was at 0.25% or below. To check for robustness, we repeat our analysis setting δ to 1.4%, 1.6%, 1.8%, 2.2%, and 2.4% and find very similar results.

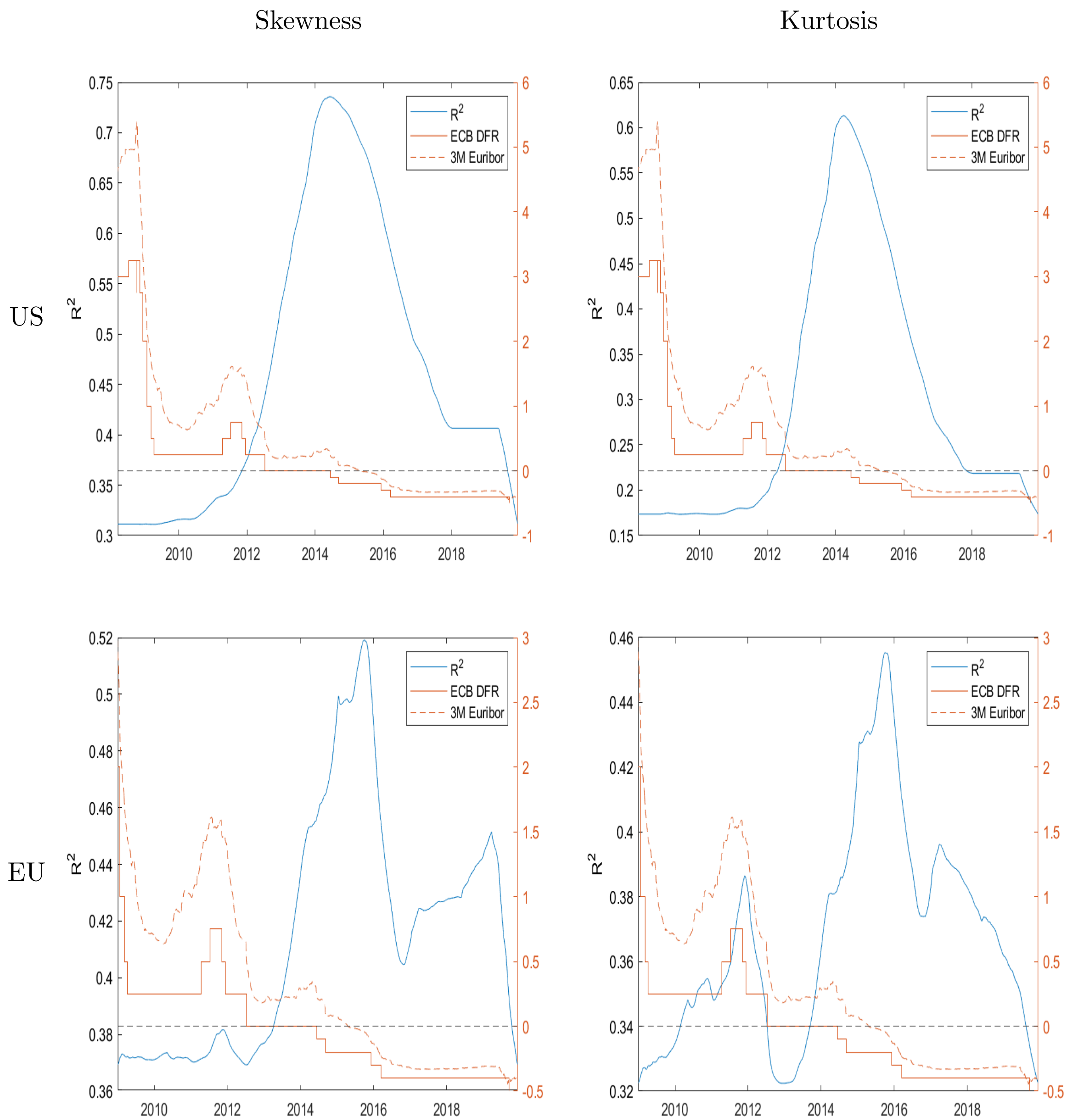
Regarding the estimated timing of the occurred break, Figure 5 shows the structural-break regression R^2s for skewness and kurtosis in both markets as a function of T_b. Our point estimates of T_b maximize R^2's over the sample. Breakdate point estimates in the different regressions range from mid-2014 in the US to September 2015 in the Eurozone. Strikingly, this period coincides with the fall of interest rates into deeply negative territory in many major economies for the first time in history. The European Central Bank (ECB) cut its deposit facility rate to -10 basis points in June 2014, followed by further rate cuts to -20 basis points in September 2014 and -30 basis points in December 2015. Denmarks Nationalbank, Swedens Riksbank, and the Swiss National Bank also all cut their policy rate to new below-zero lows during the same period.[4] Short-term market rates in the Eurozone such as 1-month Euribor started to drop below zero by the end of the first quarter 2015 and continued to fall to about -20 basis points by the end of the same year. Hence the impact of a lower bound on interest-rate distributions seems to have undergone a substantial structural change just when markets experienced the clear violation of the previously widely held zero-lower-bound assumption.

Note that although all estimated breakdates fall into a relatively short period of time they are

[4]Sweden and Denmark already experienced short periods of somewhat negative policy rates in 2009 and 2012, respectively.

unlikely to be directly associated with any particular single-day event. For the sake of simplicity and parsimony, we focused on the detection of an immediate break and did not model a smooth transition of parameter values. On a daily frequency, this approach presents a simple and straightforward approximation to locate the period over which parameters might have adjusted. It does not seem unreasonable, however, that markets updated their lower-bound beliefs at least somewhat gradually over a short period, instead of abruptly from one day to the other. In fact, it appears unrealistic to identify a single-day event at which one would expect markets to suddenly fully revise their lower-bound beliefs. The initial policy rate cut below zero by the ECB was largely anticipated by market participants according to Bloomberg survey forecasts. Also, the use of negative policy rates has been publicly discussed with increasing intensity preceding the actual announcement. Some of the subsequent cuts, such as the ECB rate cut in September 2014, were more of a surprise according to surveys. Still it remains unclear why a surprise policy rate cut would immediately change markets lower-bound expectations. The theoretical justification of a lower bound assumes that market participants start holding cash instead of lending at interest rates below corresponding opportunity costs. In reality, the cost of holding cash might differ across market participants, making this mechanism a somewhat smooth transition as interest rates approach their lower bound. One would imagine the market to require some time to learn whether this mechanism is actually in place in line with the assumed level of the lower bound. Our approach abstracts from these nuances and puristically captures the period over which markets adapted their lower-bound beliefs and incorporated them gradually into option prices. The absence of an immediate break on a daily frequency partly explains the slight differences in breakdate point estimates between the US and the Eurozone. While US short rates remained approximately unchanged over the break period, Eurozone short rates fell and partly offset the effect of a gradual downward revision of the lower bound.

Figure 5: Break-date estimation



The figure plots R^2s of the structural-break-regressions (4) depending on the date of the structural break T_b. Our least-square estimates for the break-date parameter T_b maximize R^2. Additionally, the deposit facility rate set by the European central bank and the three-month Euribor rate are plotted.

Tables 4, 5, 6, and 7 display results of the structural-break regressions, evaluated separately for skewness and kurtosis in each market. The regressions are run for each maturity individually and the row "Total" shows the results from the regression pooling all maturities together. Regressions across all maturities are uniformly evaluated at the same break-date T_b as estimated previously on the full sample. For each regression, the tables report estimates for the coefficients α, β_0 and β_1, their Newey-West standard errors in parentheses, the regression-R^2, and the number of observations. Furthermore, column F shows the SupF-test statistic evaluating the significance of the estimated break for each individual maturity.

The estimated break for skewness is (highly) significant for all maturities in both markets. Also, the break in conditional kurtosis is significant for all maturities in the US and all but two maturities in the Eurozone. This indicates strong evidence for a structural break in distribution moments conditional on low interest rates. We conclude that the impact of a lower bound on the distribution of interest rates has undergone significant change throughout the sample, implying a corresponding change in the market-implied location of the lower bound. Hence, to understand the strength of a lower-bound impact, and particularly the fixed lower-bound inconsistent conditional distributions of negative interest rates in the Eurozone, it will be crucial to account for this apparently occurred shift. Notably, the structural break tests are not only significant for the Eurozone data, where conditional distributions of negative rates already pointed to such a break, but also for the US data. This documents a completely novel finding, since a change in the location of the lower bound in the US has not been considered yet in the literature.

Turning to the model estimates, we start by comparing the average conditional distributions near the lower bound, given by $\hat{\alpha} + \widehat{\beta_0}$ for the pre-break-period and by $\hat{\alpha} + \widehat{\beta_1}$ for the post-break-period, to the theoretical floor $\underline{\mathcal{S}}$ and $\underline{\mathcal{K}}$. For the pre-break period, average skewness conditional on rates being below 2% is 2.86 for the US and 2.07 for the Eurozone. This is consistent with interest rates being on average only about $0.32 - 0.40$ standard deviations away from the lower bound. For the post-break period, average conditional skewness is substantially lower for both markets, summing to 0.31 and 0.97 respectively. This implies a lower-bound location of at minimum more than 0.63-0.86 standard deviations below observed interest rates. Hence the identified break in conditional skewness is consistent with a corresponding downward shift of the lower bound in both markets.

Table 4: Structural break regression: US skewness

Maturity (in months)	α	β_0	β_1	F	R^2	Observations
0-3	-0.16	2.77***	-0.06	162.4***	0.697	897
	(-0.48)	(5)	(-0.16)			
3-6	-0.68**	3.72***	0.66*	105.74***	0.846	2672
	(-2.28)	(9.28)	(1.79)			
6-9	-0.59**	3.63***	0.77**	74.68***	0.84	3298
	(-2.41)	(10.12)	(2.16)			
9-12	-0.49**	3.43***	0.78**	43.76***	0.823	3516
	(-2.2)	(8.78)	(2.29)			
12-15	-0.42*	3.31***	0.75**	37.74***	0.8	3638
	(-1.92)	(7.65)	(2.44)			
15-18	-0.22	3.07***	0.59**	32.96***	0.752	3702
	(-0.94)	(5.93)	(2.03)			
18-21	0.01	3.04***	0.41	41.42***	0.736	3768
	(0.03)	(5.9)	(1.29)			
21-24	0.1	2.96***	0.34	39.49***	0.746	3686
	(0.38)	(6.15)	(1.11)			
24-27	-0.18	3.01***	0.64**	27.25***	0.788	2151
	(-0.7)	(6.23)	(2.42)			
27-30	0.06	2.43***	0.34	39.03***	0.769	2093
	(0.21)	(5.25)	(1.31)			
30-33	0.29	2.18***	0.06	52.3***	0.713	2042
	(0.96)	(4.89)	(0.21)			
33-36	0.45	1.64***	-0.09	95.04***	0.583	2011
	(1.63)	(5.46)	(-0.35)			
Total	-0.01	2.87	0.32		0.74	37646

This table shows the results from the structural-break regressions for skewness in the US. Regressions are run for each maturity individually and the row "Total" reports regression results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (16.06.2014) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to 252 lags (i.e. one year) using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to assess significance of the SupF-Test are taken from Bai and Perron (1998).

Table 5: Structural break regression: EU skewness

Maturity (in months)	α	β_0	β_1	F	R^2	Observations
0-3	-0.14	1.35^{***}	0.21^{*}	41.65^{***}	0.331	1165
	(-1.19)	(5.98)	(1.8)			
3-6	-0.27^{**}	1.93^{***}	0.52^{***}	49.49^{***}	0.567	2665
	(-2.22)	(8.1)	(4.15)			
6-9	-0.11	2.06^{***}	0.74^{***}	15.88^{***}	0.584	3172
	(-0.92)	(7.89)	(2.72)			
9-12	0.03	2.14^{***}	0.9^{**}	8.47^{*}	0.601	3511
	(0.2)	(7.53)	(2.33)			
12-15	0.27^{*}	1.95^{***}	0.78^{**}	11.83^{**}	0.597	3814
	(1.76)	(6.66)	(2.54)			
15-18	0.47^{**}	1.86^{***}	0.58^{*}	18.08^{***}	0.607	3896
	(2.41)	(5.86)	(1.93)			
18-21	0.56^{***}	1.83^{***}	0.59^{**}	23.51^{***}	0.639	3898
	(2.96)	(6.28)	(2.12)			
21-24	0.66^{***}	1.62^{***}	0.52^{**}	24.52^{***}	0.646	3580
	(4.12)	(6.82)	(2.01)			
Total	0.3	1.77	0.67		0.52	25768

This table shows the results from the structural-break regressions for skewness in the Eurozone. Regressions are run for each maturity individually and the row "Total" reports regression results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (30.09.2015) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to 252 lags (i.e. one year) using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to assess significance of the SupF-Test are taken from Bai and Perron (1998).

Table 6: Structural break regression: US kurtosis

Maturity (in months)	α	β_0	β_1	F	R^2	Observations
0-3	5.34***	9.26***	-1.04***	64.42***	0.621	897
	(17.88)	(6.92)	(-3.3)			
3-6	5.37***	11.01***	-1.31***	264.92***	0.78	2672
	(16.32)	(15.52)	(-3.18)			
6-9	4.81***	11.48***	-0.3	48.58***	0.634	3298
	(13.59)	(6.11)	(-0.4)			
9-12	4.54***	10.89***	-0.08	20.55***	0.603	3516
	(13.86)	(4.48)	(-0.11)			
12-15	4.57***	11.38***	-0.16	15.69***	0.525	3638
	(19.33)	(3.88)	(-0.25)			
15-18	4.37***	11.65***	-0.21	15.01***	0.574	3702
	(12.99)	(3.68)	(-0.37)			
18-21	4.36***	13.98***	-0.29	21.14***	0.636	3768
	(12.91)	(4.39)	(-0.55)			
21-24	4.28***	13.88***	-0.27	20.12***	0.697	3686
	(12.81)	(4.37)	(-0.53)			
24-27	4.34***	12.08***	-0.41	17.18***	0.607	2151
	(45.08)	(4.13)	(-1.23)			
27-30	4.27***	8.58***	-0.45	23.87***	0.664	2093
	(64.9)	(4.79)	(-1.52)			
30-33	4.29***	8.1***	-0.65**	23.98***	0.602	2042
	(27.51)	(4.44)	(-2.17)			
33-36	4.4***	5.13***	-0.73**	78.64***	0.542	2011
	(17.34)	(7.52)	(-2.17)			
Total	4.44	11.31	-0.3		0.61	37646

This table shows the results from the structural-break regressions for kurtosis in the US. Regressions are run for each maturity individually and the row "Total" reports regression results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (21.03.2014) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to 252 lags (i.e. one year) using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to assess significance of the SupF-Test are taken from Bai and Perron (1998).

Table 7: Structural break regression: EU kurtosis

Maturity (in months)	α	β_0	β_1	F	R^2	Observations
0-3	5.5^{***}	2.73^{***}	0.44^{**}	10.97^{**}	0.134	1165
	(28)	(3.51)	(2.24)			
3-6	4.32^{***}	5.36^{***}	1.65^{***}	8.52^{*}	0.296	2665
	(17.43)	(4.22)	(4.48)			
6-9	3.8^{***}	7.07^{***}	2.92^{***}	6.31	0.378	3172
	(20.1)	(4.86)	(3.91)			
9-12	3.61^{***}	8.19^{***}	3.38^{***}	6.88	0.454	3511
	(27.35)	(5.02)	(3.99)			
12-15	3.91^{***}	7.73^{***}	2.88^{***}	10.45^{**}	0.515	3814
	(25.19)	(5.33)	(5.1)			
15-18	4.19^{***}	7.68^{***}	2.47^{***}	14.82^{***}	0.565	3896
	(12.91)	(5.48)	(3.87)			
18-21	4.29^{***}	7.64^{***}	2.45^{***}	18.47^{***}	0.599	3898
	(14.05)	(6.25)	(3.9)			
21-24	4.19^{***}	6.62^{***}	2.48^{***}	20.91^{***}	0.607	3580
	(15.88)	(7.4)	(3.74)			
Total	4.13	6.88	2.59		0.46	25768

This table shows the results from the structural-break regressions for kurtosis in the Eurozone. Regressions are run for each maturity individually and the row "Total" reports regression results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (05.10.2015) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to 252 lags (i.e. one year) using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to assess significance of the SupF-Test are taken from Bai and Perron (1998).

Conditional kurtosis for the pre-break period is on average 15.75 in the US (11.01 in the Eurozone), which relates to a minimal distance to a lower bound of about 0.25 (0.29) standard deviations. For the post-break period, conditional kurtosis is much lower (4.13 for the US and 6.72 for the Eurozone), suggesting distribution tails again closer to normal. Compared to the theoretical floor $\underline{\mathcal{K}}$, the average minimal distance to the lower bound increases for the post-break period to about 0.36-0.45 standard deviations.

To be consistent with an impact from a close lower bound, we further expect distribution moments conditional on low interest rates to significantly differ from values observed for higher interest rates. For skewness, estimates of the pre-break parameter $\widehat{\beta}_0$ are all positive and highly significant for every single maturity in both markets. Particularly for the US, the impact on skewness is substantial with an estimated difference of 2.87. In the Eurozone, the absolute value of parameter estimates is somewhat more muted. Post-break parameter estimates $\widehat{\beta}_1$ are in all cases much smaller, in many cases insignificant and sometimes even slightly negative. In summary, these results indicate tightly constrained interest-rate distributions leading to substantially above average skewness levels in the pre-break period. In the post-break period, evidence for any impact on skewness is much weaker. At its best, it is quantitatively only very small and only a fraction of the pre-break impact.

Results for kurtosis confirm these finding. $\widehat{\beta_0}$ estimates are all positive and highly significant. Estimates are also quantitatively large with the effect again more pronounced in US data. Hence distributions conditional on low interest rates in the pre-break period have much heavier tails than normally. For the post-break period, estimates for $\widehat{\beta_1}$ are negative for all maturities in the US and mostly insignificant. In the Eurozone, estimates are still positive and significant but quantitatively two to three times smaller than the pre-break estimates. This confirms the strongly reduced importance of an impact of a lower bound on interest-rate distributions after the occurred break.

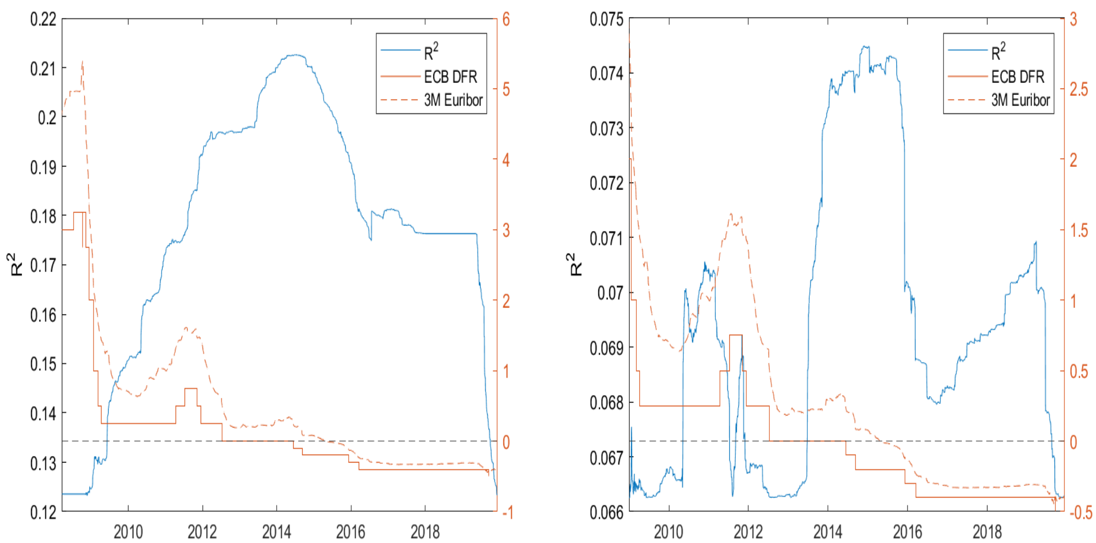
Finally, we test whether a downward revision of the lower bound as indicated by skewness and kurtosis data is also reflected by a corresponding change in conditional level-volatility dependence. To this end, we extend the level-volatility regression (3) to account for a relation conditional on low interest rates subject to a break:

$$\Delta Vol_{t,T} = \alpha + \beta \Delta f_{t,T} + \gamma_0 \Delta f_{t,T} I_{\left(f_{t,T}<2\right)} I_{(t<T_b)} + \gamma_1 \Delta f_{t,T} I_{\left(f_{t,T}<2\right)} I_{(t\geq T_b)} + \epsilon_{t,T} \tag{5}$$

Here, γ_0 and γ_1 estimate the pre- and post-break impact of a lower bound on level-volatility dependence and β estimates the relationship without a lower-bound impact. Figure 6 shows that break-date estimates for this regression fall into the range of breaks identified for skewness and kurtosis in both markets. Column F in Tables 8 and 9 indicate that these breaks are (highly) significant for all except one maturity in the US-market. Hence the identified break in conditional skewness and kurtosis coincides with a significant change in the conditional level-volatility relation. β's are small in magnitude and only for few maturities significantly different from zero, indicating little level-dependence of volatility apart from the impact of a lower bound. In the pre-break period, the impact on level-volatility dependence of conditioning on interest rates close to zero is highly significant and positive in both markets and for all maturities (γ_0). This confirms the finding of a tightly binding lower bound in the pre-break period. For the post-break period, γ_1 estimates are slightly negative on average as well as for most maturities, and mostly insignificant. The structural change in conditional level-volatility is therefore also consistent with a strong downward revision of the lower bound around the introduction of negative interest rates.

Figure 6: Break-date estimation: level-volatility dependence

US EU



The figure plots R^2s of the structural-break regressions (5) depending on the date of the structural break T_b. Our least-square-estimates for the break-date parameter T_b maximize R^2. Additionally, the deposit facility rate set by the European central bank and the three-month Euribor rate are plotted.

Table 8: Structural break regression: US level-dependence of volatility

Maturity (in months)	α	β	γ_0	γ_1	F	R^2	Observations
0-3	0.0019	0.47^{***}	0.78^{**}	-1^{***}	38.34^{***}	0.192	897
	(0.67)	(2.62)	(2.38)	(-4.6)			
3-6	0.0003	-0.01	1.05^{***}	-0.19^{**}	194.71^{***}	0.33	2672
	(0.39)	(-0.19)	(11.47)	(-2.25)			
6-9	-0.0001	-0.02	0.79^{***}	-0.07	202.12^{***}	0.381	3298
	(-0.28)	(-0.6)	(13.17)	(-1.29)			
9-12	-0.0003	-0.02	0.61^{***}	-0.01	192.69^{***}	0.415	3516
	(-0.97)	(-0.89)	(14.03)	(-0.36)			
12-15	-0.0003	0.01	0.43^{***}	-0.02	253.3^{***}	0.315	3638
	(-0.79)	(0.26)	(10.79)	(-0.37)			
15-18	-0.0003	0.02	0.34^{***}	0.01	163.64^{***}	0.257	3702
	(-1.13)	(0.71)	(11.28)	(0.31)			
18-21	-0.0002	0.05^{**}	0.29^{***}	-0.01	188.7^{***}	0.242	3768
	(-0.88)	(2.04)	(10.45)	(-0.39)			
21-24	-0.0001	0.08^{***}	0.26^{***}	-0.04	251.07^{***}	0.289	3668
	(-0.55)	(4)	(10.38)	(-1.58)			
24-27	-0.0005^{***}	-0.03	0.36^{***}	0.06^{**}	305.87^{***}	0.418	2141
	(-2.6)	(-1.1)	(12.45)	(2.22)			
27-30	-0.0005^{**}	0.01	0.31^{***}	0.02	207.56^{***}	0.456	2092
	(-2.46)	(0.39)	(12.45)	(0.96)			
30-33	-0.0004^{*}	0.05^{***}	0.24^{***}	0.09	5.74	0.49	2041
	(-1.73)	(3.66)	(11.68)	(1.43)			
33-36	0.0001	0.09^{***}	0.16^{***}	-0.07^{***}	76.46^{***}	0.271	2002
	(0.31)	(6.88)	(6.41)	(-3.75)			
Total	-0.0003	0.05	0.44	-0.02		0.21	37580

This table shows the results from the structural-break regressions for level-volatility dependence in the US. Regressions are run for each maturity individually and the row "Total" reports regression-results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (08.07.2014) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to two lags using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to asses significance of the SupF-Test are taken from Bai and Perron (1998)

Table 9: Structural break regression: EU level-dependence of volatility

Maturity (in months)	α	β	γ_0	γ_1	F	R^2	Observations
0-3	0.0004	-0.36**	0.92***	-0.11	11.14**	0.062	1165
	(0.24)	(-2.5)	(3.48)	(-0.43)			
3-6	-0.0005	-0.19***	0.59***	0.2	8.2*	0.101	2665
	(-0.9)	(-3.08)	(6.02)	(1.61)			
6-9	-0.0005	-0.06	0.38***	-0.05	23.24***	0.106	3172
	(-1.13)	(-1.59)	(6.71)	(-0.61)			
9-12	-0.0006*	-0.01	0.28***	-0.06	23.36***	0.119	3512
	(-1.79)	(-0.25)	(6.91)	(-0.87)			
12-15	-0.0004	0.04*	0.21***	-0.1	25.63***	0.127	3815
	(-1.31)	(1.76)	(6.57)	(-1.62)			
15-18	-0.0004	0.09***	0.13***	-0.1**	19.82***	0.121	3897
	(-1.63)	(4.37)	(3.73)	(-2.04)			
18-21	-0.0003	0.1***	0.16***	-0.08*	31.06***	0.159	3898
	(-1.09)	(4.72)	(5.62)	(-1.87)			
21-24	0	0.1***	0.13***	-0.05	13**	0.135	3537
	(-0.04)	(4.76)	(4.95)	(-0.99)			
Total	-0.0003	0.03	0.26	-0.05		0.07	25711

This table shows the results from the structural-break regressions for level-volatility dependence in the Eurozone. Regressions are run for each maturity individually and the row "Total" reports regression-results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (24.11.2014) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to two lags using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to asses significance of the SupF-Test are taken from Bai and Perron (1998)

5 Risk Premia

Our analysis up to here relies on option-implied distributions. Their forward-looking nature allows us to most directly capture markets' *ex ante* lower bound beliefs and provides us with reliable high frequency estimates. However, estimated distribution characteristics pertain to risk-neutral distributions. Therefore, one might be concerned as to what degree our conclusions could be driven by time variation in risk premia instead of physical distribution characteristics.

Even though the physical ($\mathbb{P}$) distribution differs from the associated risk-neutral ($\mathbb{Q}$) distribution by risk premia, the two measures also have some common features. By definition, the two measures are equivalent and thus they coincide in which events have measure zero. Therefore, any lower bound of the physical interest-rate distribution (measure $\mathbb{P}$) is concomitantly the lower bound of the associated risk-neutral interest-rate distribution (measure $\mathbb{Q}$) and vice versa. Consequently, the maximum lower bound we derive from option-implied moments (see Figure 3) is generally valid, i.e., it applies to any measure, be it $\mathbb{Q}$ or $\mathbb{P}$. If we were able to determine the lower bound explicitly, we could safely rely on option-implied moments to make general statements about the lower bound. However, several of our statements rely on a relationship between the distance to the lower bound and the minimum skewness and kurtosis (see Figure 1). While this relationship also holds both under $\mathbb{P}$ and $\mathbb{Q}$, the "distance to the lower bound" d depends on the respective measure. Strictly speaking, option-implied moments thus only allow us to exclude a "tightly binding lower bound" as measured in terms of risk-neutral interest rate expectations and volatilities. Additionally, one might be worried that the elevated levels of implied skewness and kurtosis of near-zero interest rates in the former part of our sample might stem from risk premia instead of being induced by a zero lower bound.

To address these concerns, we first estimate the conditional moments of the physical distribution. Neuberger (2012) and Bae and Lee (2021) construct unbiased measures for realized skewness and kurtosis. We follow their approach to calculate realized skewness and kurtosis at a monthly frequency. Neuberger (2012) shows that skewness at longer horizons originates from two sources: skewness of high frequency price changes and the covariation between high frequency price changes and shocks to (implied) variance. Consequently, our estimation of realized skewness relies on daily changes in the futures rate as well as daily changes in implied variance (as estimated in Equation 18) to account for both sources of skewness of the terminal interest-rate distribution. Similarly, Bae and Lee (2021) show that an unbiased estimate of the kurtosis of a longer horizon distribution needs to be constructed from the sub-period increments of not only prices but also the variance and the expected third raw moment. Hence, our estimates of realized kurtosis additionally require daily changes in the expected third raw moment, which we directly derive from option-implied skewness (Equation 20) and variance (Equation 18).
Figure A.1 in the appendix plots the time-series of realized moments. The realized moments are considerably more volatile than their option-implied counterparts and display some erratic behavior. However, although realized skewness and kurtosis appear much noisier, they closely resemble the low-frequency variation of implied moments. In particular, they are clearly elevated during the period of near-zero interest-rates and experience a sharp drop around 2014/2015. Taken together, the trajectories of the physical moments lead to similar conclusions about the importance and stability of market expectations about a lower bound on interest rates.

We next replicate our formal analysis replacing option-implied moments with realized moments. The results are summarized in Appendix A.5. Table A.3 and A.4 in the appendix replicate the results on level-dependent distribution characteristics from Table 1 and 2. Again, we see that the interest rate level alone is not able to explain the time variation in skewness and kurtosis. As a final robustness check, we also test our results on a potential shift in lower-bound views based on realized moments. Figure A.2 shows that break-date estimates based on realized moments quite closely match option-based estimates. Tables A.5 - A.8 replicate the structural break regression results. Again, our main results remain quite robust to the change in measure. Overall, our statements based on the distance to the lower bound also appear to be unaffected by time-varying risk premia.

6 Conclusion

This paper studies market participants' anticipation of a lower bound on interest rates. To this end we analyze the moments of option-implied interest-rate distributions in the US and the Eurozone.

We find that the lower bound temporarily had an important impact, as the distributions of interest rates close to zero have been highly asymmetric. However, our results suggest that market participants significantly changed their lower-bound views over time. Using a structural change regression, we present evidence for a large structural break of the distribution characteristics of low interest rates, consistent with a marked downward revision of markets' lower-bound beliefs. Prior to the occurred break we document strong evidence of a substantial impact of a lower bound on interest-rate distributions. Pre-break distributions of low interest rates exhibit high positive skewness and kurtosis and strong positive level-dependence. After the break, any lower-bound impact is at best quantitatively small and evidence for it becomes inconclusive. We consistently estimate this break to have occurred within the period when interest rates in many major economies fell into deeply negative territory for the first time in history in 2014/2015. These results point to the updating of investors' long-held zero lower bound beliefs in light of the negative interest rate experience.

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A Appendix

A.1 Proof of Proposition 1

Proof. Symmetry implies that P_t has support only on $[LB, 2\mu - LB]$. Popoviciu's inequality (Popoviciu, 1935) for probability distributions bounded by a and b then immediately gives $\sigma^2 \leq \frac{(b-a)^2}{4} = (\mu - LB)^2$. Hence, the volatility for a symmetric distribution is bounded by $\sigma \leq (\mu - LB) = \overline{\sigma}$ and the distance to lower bound by $d \geq \frac{(\mu - LB)}{\overline{\sigma}} = 1$. $\square$

A.2 Proof of Proposition 2

Proof. Problems (1),(2) can be formulated as the general moment problem

$$\inf_{P \in \mathcal{P}} \int_{LB}^{\infty} H(L_T) dP = \inf_{P \in \mathcal{P}} \int_{LB}^{\infty} \left(\frac{L_T - \mu}{\sigma} \right)^z dP, z \in \{3, 4\} \tag{6}$$

s.t.

$$\int_{LB}^{\infty} L_T dP = \mu \tag{7}$$

$$\int_{LB}^{\infty} L_T^2 dP = \mu^2 + \sigma^2 \tag{8}$$

Karr (1983) studies properties of generalized moment problems like (6). He shows that optimal solutions are obtained at extreme points of $\mathcal{P}$, which are probability measures with finite support on $L \leq N + 1 = 3$ distinct points (Theorem 2.1, Karr (1983)). If there exists the continuous 1^{st} derivative of H, H', and H' is strictly convex on $[LB, \infty)$, an insight from Krein and Nudelman (1977) (Chapter 4, Theorem 1.1) applies and further simplifies the problem: in this case, extremal solutions to (6) are obtained for distributions with only two distinct points of support $\{x_1, x_2\}$, where $x_1 = LB$ and x_2, as well as the probability weights $\{p_1, p_2\}$, are uniquely determined by conditions (7),(8) and $\int_{LB}^{\infty} dP = 1$.

For $\underline{S}$ then immediately follows

$$\underline{S} = \frac{1 - d^2}{d}$$

which is obtained for the two-point-support distribution $\{\{x_1, x_2\}, \{p_1, p_2\}\}$ with

$$\begin{aligned} x_1 &= LB \\ x_2 &= \frac{\mu^2 + \sigma^2 - LB\mu}{\mu - LB} \\ p_1 &= \frac{\sigma^2}{(\mu - LB)^2 + \sigma^2} = \frac{1}{d^2 + 1} \\ p_2 &= \frac{d^2}{d^2 + 1} \end{aligned}$$

For $\underline{\mathcal{K}}$ we consider the program

$$\min_{\{x_i,p_i\}} \sum_{i=1}^{3} \left(\frac{x_i-\mu}{\sigma}\right)^4 p_i \tag{9}$$

s.t.

$$\sum_{i=1}^{3} p_i = 1 \tag{10}$$

$$\sum_{i=1}^{3} x_i p_i = \mu \tag{11}$$

$$\sum_{i=1}^{3} x_i^2 p_i = \mu^2 + \sigma^2 \tag{12}$$

$$p_1, p_2, p_3 \geq 0 \tag{13}$$

$$LB \leq x_1 < x_2 < x_3 \tag{14}$$

The program can be solved with the Lagrange Method. Note that the problem is obviously only feasible if $x_1 \in [LB, \mu)$ and $x_3 > \mu$. For $\mu - \sigma \geq LB(d \geq 1)$ the optimal solution with inactive inequality constraint $x_1 \geq LB$ is given by the two-point-support distribution with $x_1 = \mu - \sigma, x_2 = \mu + \sigma, p_1 = p_2 = 0.5$ and $\underline{\mathcal{K}} = 1$.

For $\mu - \sigma < LB(d < 1)$, the optimal constrained solution is the same discrete distribution as derived for $\underline{\mathcal{S}}$. Kurtosis for this solution is given by $\underline{\mathcal{K}} = \frac{1+d^6}{d^4+d^2}$

□

A.3 Dataset

Our derivatives dataset consists of daily settlement prices of futures and options on the three-month USD-LIBOR rate (Eurodollar futures) and the three-month Euribor rate trading at CME and at ICE Futures Europe. Option maturities are available at monthly frequency, but we limit our analysis to quarterly expiries. These options are most liquid and much easier to value as their expirations coincide with those of the underlying futures. Conveniently, they are in effect options on the underlying reference rate itself. Quarterly Eurodollar (Euribor) options have maturities of up to four (two) years, providing us with 16 (8) points across the short-end of the yield curve at any given day. We further limit our data to out-of-the-money (OTM) options, which are typically more liquid than in-the-money options (see e.g. Puigvert-Gutiérrez and de Vincent-Humphreys (2012)). This gives us unique option prices for each strike-maturity combination as required by our empirical methodology.

Finally, we apply some additional standard filters to our options sample. We exclude option prices which are negative or below the minimum tick-size, and we exclude prices at the last trading day. Further, option prices are screened to meet the basic arbitrage considerations. After applying all filters, we estimate option-implied moments on a given day for all maturities with at least two put options and two call options. Table A.2 provides details on the composition of our option sample.

While Eurodollar and Euribor options are conceptually very similar, we point out one difference which affects their pricing and the calculation of implied moments: Unlike Eurodollar options, Euribor options are subject to futures-style margining. That is, the premium for Euribor options is not paid upfront at the time of purchase. Instead, option positions are marked-to-market giving rise to daily variation margin flows. This simplifies pricing of these options as it no longer involves discounting or an early-exercise premium (see Lieu (1990) and Chen and Scott (1993)).

Table A.1: Trading Volume

	Eurodollar		Euribor		10-Yr US Treasury Note	
	Futures	Options	Futures	Options	Futures	Options
Open interest	10,940,505	36,266,803	3,821,640	4,653,484	3,623,839	3,116,891
Volume	36,492,894	15,285,898	11,283,635	1,033,095	28,427,186	15,066,820
Average daily volume	1,737,757	727,900	537,316	49,195	1,353,676	717,468

This table shows trading volume and open interest as of December 2019 for the future and option contracts used in our study. The last two columns additionaly provide trading statistics for 10-Year US treasury notes for comparison. Eurodollar contracts and 10-Year US treasury futures and options trade at CME, Euribor contracts trade at ICE Futures Europe. Data comes directly from CME and ICE Futures Europe.

Table A.2: Sample composition

Maturity (in months)	Moneyness					
	0.99-1	0.98-0.99	0.97-0.98	0.96-0.97	≤ 0.96	Total
Panel A: US						
0-3	7,677	894	191	42	0	8,804
3-6	24,601	5,000	967	225	40	30,833
6-9	35,358	10,279	2,463	839	150	49,089
9-12	43,308	15,902	4,807	1,527	491	66,035
12-15	48,057	22,065	7,794	2,452	1,804	82,172
15-18	50,835	29,139	10,745	3,597	3,146	97,462
18-21	52,010	34,452	13,159	4,609	4,559	108,789
21-24	49,596	34,589	12,563	5,003	4,645	106,396
24-27	31,783	23,971	8,514	2,791	2,113	69,172
27-30	31,212	25,395	10,266	3,554	2,708	73,135
30-33	30,588	25,890	11,749	4,175	3,584	75,986
33-36	28,825	24,746	12,110	5,052	4,033	74,766
36-39	22,511	21,401	10,570	5,097	3,557	63,136
39-42	17,939	16,865	8,757	4,662	3,855	52,078
42-45	14,137	13,212	6,975	4,273	3,988	42,585
45-48	10,883	9,758	5,267	3,556	3,644	33,108
Total	499,632	313,817	127,045	51,557	42,443	1,034,494
Panel B: EU						
0-3	8,799	449	16	2	0	9,266
3-6	24,351	2,783	82	6	0	27,222
6-9	34,219	7,735	1,093	25	0	43,072
9-12	42,165	12,610	3,896	410	6	59,087
12-15	35,389	12,020	5,413	2,012	349	55,183
15-18	38,793	15,338	7,045	3,431	949	65,556
18-21	41,285	19,248	8,656	4,324	1,895	75,408
21-24	39,395	21,427	9,090	3,900	1,614	75,426
Total	264,396	91,610	35,291	14,110	4,813	410,220

This table provides details about the composition of our sample. For each combination of option maturity and moneyness, we count the number of available daily option prices in our sample running from June 2006 (May 2005) to December 2019 for the US (Eurozone) market. Moneyness is calculated for call (put) options as the ratio of futures (strike) price to strike (futures) price. Eurodollar contracts and 10-Year US treasury futures and options trade at CME, Euribor contracts trade at ICE Futures Europe. Data comes from Thomson Reuters Datastream.

A.4 Model-free estimation of option-implied interest rate moments

Consider a European-style call option with maturity T and strike K. Its value at time t is given by

$$C_t(K) = \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^T r_t dt} (L_T - K)^+ \right] \tag{15}$$

$$= B_{t,T} \mathbb{E}_t^{\mathbb{Q}_T} \left[(L_T - K)^+ \right] \tag{16}$$

where $\mathbb{Q}$ denotes the risk-neutral measure, $\mathbb{Q}_T$ is the T-forward measure, and $B_{t,T}$ is the price of a default-free zero-coupon bond maturing in T.

For options on Euribor, valuation simplifies due to futures style margining: the discounting term drops from (15) and early exercise is never optimal, justifying the above treatment as European options. This result has been shown by Lieu (1990) and Chen and Scott (1993). Valuation for Euribor options therefore narrows down to just taking the expectation of the terminal payoff under the risk-neutral measure:

$$C_t(K) = \mathbb{E}_t^{\mathbb{Q}} \left[(L_T - K)^+ \right]$$

For Eurodollar options, the use of the $\mathbb{Q}_T$ measure becomes necessary to account for stochastic interest rates (Equation (16)). The $\mathbb{Q}_T$ measure is closely related to the $\mathbb{Q}$ measure but uses a zero-coupon bond with maturity T as numeraire instead of the money-market cash-account. Furthermore, in case of Eurodollar options, the European style valuation in (16) constitutes an approximation, neglecting the value of early exercise. However, based on existing evidence for Eurodollar options, we expect this effect to be negligible (See Flesaker (1993b), Flesaker (1993a) and Cakici and Zhu (2001)).

Following Bakshi and Madan (2000), for any fixed Z, one can write any twice differentiable payoff function of L_T, $g(L_T)$ as:

$$g(L_T) = g(Z) + g'(Z)(L_T - Z) + \int_Z^{\infty} g''(K)[L_T - K]^+ dK + \int_0^Z g''(K)[K - L_T]^+ dK \tag{17}$$

Setting $Z = \mathbb{E}_t (L_T)$ and taking expectations under the appropriate measure one can obtain expressions for conditional moments in terms of out-of-the-money options.

Bakshi et al. (2003) use this result to derive formulas for the risk-neutral moments pertaining to the *log-return* distribution of the underlying. We are however interested in the distribution of the *level* L_T of future interest rates, which directly correspond to our derivations in Section 2. Formulas for the central moments of the level of interest rates at the time horizon of the option expiry can be obtained based on (17) by setting $g(L_T)$ to the corresponding moment.

For Eurodollar options we obtain

$$
\begin{aligned}
Var^{\mathbb{Q}_T}_{t,T}(L_T) &= \mathbb{E}^{\mathbb{Q}_T}\left[(L_T - F_{t,T})^2\right] \\
&= \frac{2}{B_{t,T}}\left[\int_{F_{t,T}}^{\infty} C(K)dK + \int_0^{F_{t,T}} P(K)dK\right] && (18) \\
Vol^{\mathbb{Q}_T}_{t,T}(L_T) &= \sqrt{Var^{\mathbb{Q}_T}} && (19) \\
Skew^{\mathbb{Q}_T}_{t}(L_T) &= \mathbb{E}^{\mathbb{Q}_T}\left[\left(\frac{L_T - F_{t,T}}{\sigma}\right)^3\right] \\
&= \frac{6}{B_{t,T}Var^{\mathbb{Q}_T}_{t}(L_T)^{\frac{3}{2}}}\left[\int_{F_{t,T}}^{\infty} (K - F_{t,T})C(K)dK + \int_0^{F_{t,T}} (K - F_{t,T})P(K)dK\right] && (20) \\
Kurt^{\mathbb{Q}_T}_{t,T}(L_T) &= \mathbb{E}^{\mathbb{Q}_T}\left[\left(\frac{L_T - F_{t,T}}{\sigma}\right)^4\right] \\
&= \frac{12}{B_{t,T}Var^{\mathbb{Q}_T}_{t}(L_T)^2}\left[\int_{F_{t,T}}^{\infty} (K - F_{t,T})^2C(K)dK + \int_0^{F_{t,T}} (K - F_{t,T})^2P(K)dK\right] && (21)
\end{aligned}
$$

where $F_{t,T}$ denotes the forward LIBOR rate. Formulas for Euribor options are analogous, only that moments are measured under $\mathbb{Q}$ and the discounting term $\frac{1}{B_{t,T}}$ drops from (18)-(21).

While evaluation of equations (18)-(21) demands a continuum of option prices, traded LIBOR options are available only for a discrete finite set of strikes. Therefore empirical implementation requires inter- and extrapolation of option prices across strikes. We follow Jiang and Tian (2005) and Carr and Wu (2009) among others to fit an interpolating function in strike implied-volatility space, which is more reliable then interpolation directly in strike-price space. To calculate (18)-(21) for a given observation date and maturity we accordingly proceed as follows: First, we translate observed settlement prices of out-of-the-money put and call options into normal implied volatilities (IV). These volatilities equate theoretical option prices under the assumption of normally distributed future LIBOR rates to observed option prices. Second, we interpolate between obtained IVs by fitting a cubic spline and extrapolate outside the lowest and highest strikes using the IV at each boundary. Third, from the fitted IVs we obtain a fine grid of strike prices and evaluate the integrals in (18)-(21) through numerical integration using Simpson's rule. This way, for each trading day we obtain estimates of $Vol_{t,T}(L_T)$, $Skew_{t,T}(L_T)$ and $Kurt_{t,T}(L_T)$ for all available quarterly expirations.

A.5 Results for realized moments

Figure A.1: Time-series of realized moments

This figure shows time-series of monthly realized skewness and kurtosis, each interpolated between adjacent maturities to a constant maturity of one year. Red lines show 12-month moving average values.

Table A.3: Conditional realized skewness

	Total	<0%	0%-2%	2%-4%	>4%
Panel A: US					
Mean	0.84	-	1.45	0.32	-0.87
t-stat	-	-	5.17	-2.74	-5.61
stdv	2.31	-	2.28	2.22	1.38
N	1727	0	981	577	169
Panel B: EU					
Mean	0.69	-0.04	1.27	0.59	0.06
t-stat	-	-3.12	4.35	-0.49	-1.48
stdv	1.78	1.85	1.54	1.51	2.24
N	1186	261	523	272	130

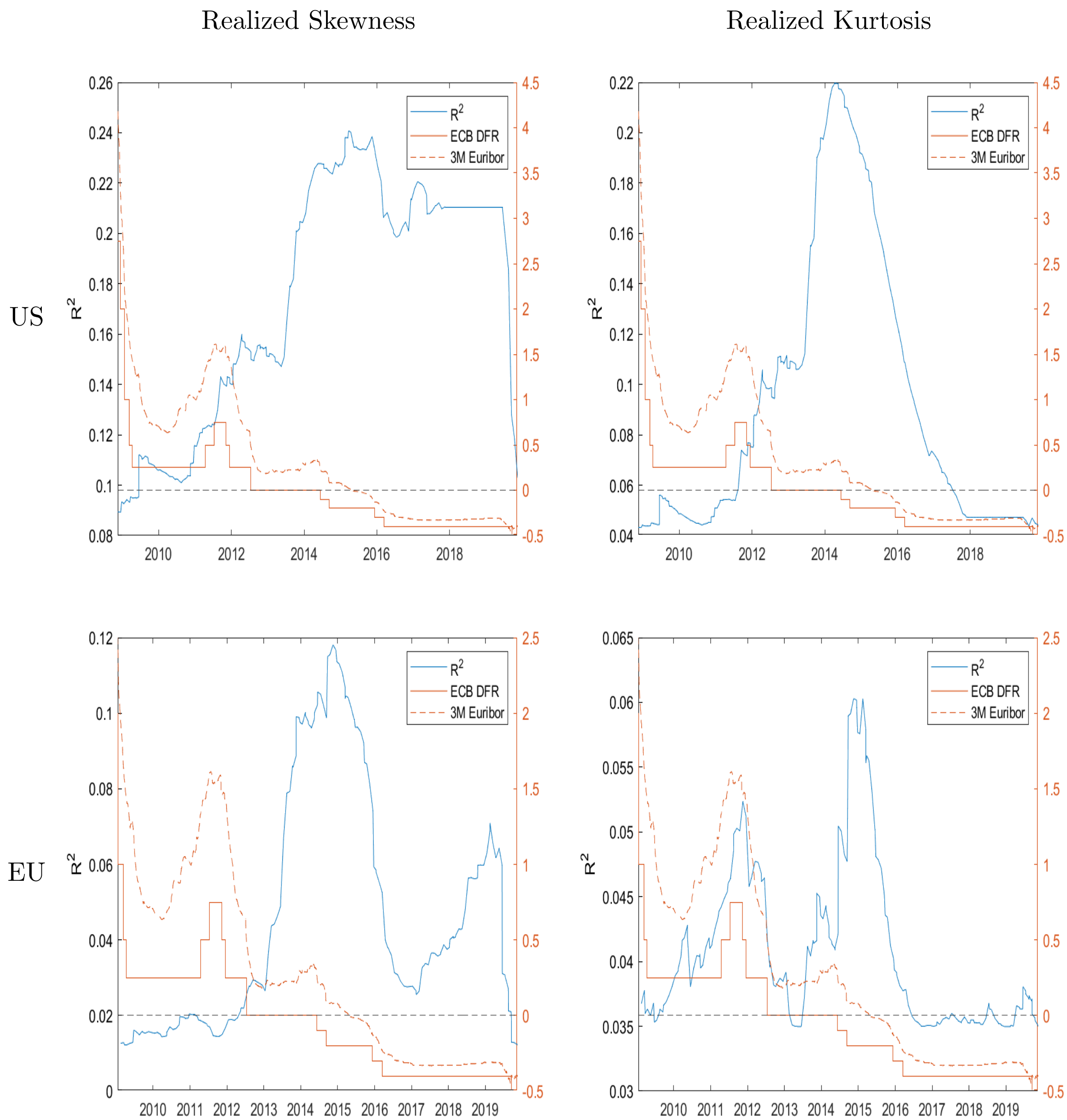
This table shows summary statistics of monthly realized skewness for the US and the Eurozone across all available maturities. The first column displays unconditional results while the second to fifth column display results conditional on the futures rate being below 0%, in the interval 0%-2%, in the interval 2%-4% and above 4%, respectively. The second row within each panel shows t-statistics of the slope coefficient from the regression $Skew_{t,T} = \alpha + \beta I_{\left(a \leq f_{t,T} < b\right)}$. t-statistics are corrected for heteroscedasticity and serial correlation up to 12 lags (i.e. one year) using the approach of Newey and West (1987).

Table A.4: Conditional realized kurtosis

	Total	<0%	0%-2%	2%-4%	>4%
Panel A: US					
Mean	9.96	-	12.34	7.31	5.12
t-stat	-	-	4.42	-3.39	-4.55
stdv	13.18	-	15.64	8.73	3.9
N	1727	0	981	577	169
Panel B: EU					
Mean	9.92	9.47	12.21	7.48	6.71
t-stat	-	-0.45	3.32	-3.04	-2.68
stdv	10.3	9.92	12.35	5.66	6.87
N	1186	261	523	272	130

This table shows summary statistics of monthly realized kurtosis for the US and the Eurozone across all available maturities. The first column displays unconditional results while the second to fifth column display results conditional on the futures rate being below 0%, in the interval 0%-2%, in the interval 2%-4% and above 4%, respectively. The second row within each panel shows t-statistics of the slope coefficient from the regression $Kurtosis_{t,T} = \alpha + \beta I_{\left(a \leq f_{t,T} < b\right)}$. t-statistics are corrected for heteroscedasticity and serial correlation up to 12 lags (i.e. one year) using the approach of Newey and West (1987).

Figure A.2: Break-date estimation realized moments



The figure plots R^2s of the structural-break-regressions (4) depending on the date of the structural break T_b. Our least-square estimates for the break-date parameter T_b maximize R^2. Additionally, the deposit facility rate set by the European central bank and the three-month Euribor rate are plotted.

Table A.5: Structural break regression: US realized skewness

Maturity (in months)	α	β_0	β_1	F	R^2	Observations
0-3	-	-	-	-	-	-
3-6	-0.86*	2.56***	0.65	19.35***	0.326	127
	(-1.77)	(5.07)	(1.24)			
6-9	-0.91**	2.77***	0.68	11.56**	0.32	148
	(-2.18)	(5.49)	(1.3)			
9-12	-0.72	2.85***	0.49	12.28**	0.3	168
	(-1.41)	(5.02)	(0.76)			
12-15	-0.53	2.73***	0.37	17.27***	0.296	165
	(-1.01)	(4.91)	(0.61)			
15-18	-0.26	2.76***	0.29	12.97**	0.249	173
	(-0.44)	(3.9)	(0.39)			
18-21	0.02	2.67***	0.14	12.62**	0.218	172
	(0.03)	(3.28)	(0.17)			
21-24	0.09	2.83***	0.1	12.97**	0.28	174
	(0.17)	(4.4)	(0.14)			
24-27	-0.73	3.19***	1.02*	8.77*	0.353	102
	(-1.24)	(4.45)	(1.83)			
27-30	-0.14	3.06***	0.41	12.61**	0.34	96
	(-0.21)	(3.82)	(0.68)			
30-33	0.56	2.81***	-0.39	18.7***	0.297	92
	(0.89)	(3.91)	(-0.58)			
33-36	0.75	1.99***	-0.55	7.89	0.199	90
	(1.3)	(3.33)	(-0.7)			
Total	0.05	2.37	-0.07		0.24	1727

This table shows the results from the structural-break regressions for monhtly realized skewness in the US. Regressions are run for each maturity individually and the row "Total" reports regression results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (24.03.2015) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to 12 lags (i.e. one year) using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to assess significance of the SupF-Test are taken from Bai and Perron (1998).

Table A.6: Structural break regression: EU realized skewness

Maturity (in months)	α	β_0	β_1	F	R^2	Observations
0-3	-	-	-	-	-	-
3-6	-0.3	1.07^{***}	0.05	15.43^{***}	0.163	122
	(-1.46)	(4.12)	(0.19)			
6-9	-0.36	1.48^{***}	0.12	49.67^{***}	0.21	143
	(-1.07)	(4.01)	(0.33)			
9-12	-0.12	1.48^{***}	-0.09	12.01^{**}	0.184	165
	(-0.32)	(3.52)	(-0.16)			
12-15	0.42	1.17^{***}	-0.45	15.2^{***}	0.165	172
	(1.18)	(2.83)	(-0.91)			
15-18	0.8^{**}	1.14^{**}	-0.74	11.15^{**}	0.14	182
	(2.03)	(2.08)	(-1.3)			
18-21	0.92^{**}	1.33^{***}	-0.77	16.03^{***}	0.139	178
	(2.49)	(2.71)	(-1.37)			
21-24	0.76^{***}	1.41^{***}	-0.47	16.53^{***}	0.202	174
	(2.82)	(3.56)	(-1.06)			
Total	0.42	1.03	-0.41		0.12	1186

This table shows the results from the structural-break regressions for monthly realized skewness in the Eurozone. Regressions are run for each maturity individually and the row "Total" reports regression results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (15.11.2014) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to 12 lags (i.e. one year) using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to assess significance of the SupF-Test are taken from Bai and Perron (1998).

Table A.7: Structural break regression: US realized kurtosis

Maturity (in months)	α	β_0	β_1	F	R^2	Observations
0-3	-	-	-	-	-	-
3-6	6.45^{***}	4.9^{***}	-2.12^{***}	21.32^{***}	0.157	127
	(13.67)	(3.43)	(-4.28)			
6-9	6.34^{***}	7.68^{***}	-1.44^{**}	20.27^{***}	0.173	148
	(10.72)	(4.16)	(-2.05)			
9-12	5.88^{***}	11.04^{***}	-1.24	19.35^{***}	0.188	168
	(6.27)	(4.04)	(-1.12)			
12-15	6.33^{***}	11.61^{***}	-1.61	27.41^{***}	0.277	165
	(5.26)	(3.95)	(-1.2)			
15-18	6.93^{***}	15.11^{***}	-2.41	26.31^{***}	0.235	173
	(3.56)	(3.31)	(-1.19)			
18-21	7.99^{***}	15.26^{***}	-3.09	45.68^{***}	0.262	172
	(3.9)	(4.23)	(-1.45)			
21-24	6.99^{***}	18.1^{***}	-2.44^{*}	52.44^{***}	0.277	174
	(5.94)	(6.67)	(-1.71)			
24-27	7.26^{***}	15.31^{***}	-3.63^{**}	108.14^{***}	0.359	102
	(3.75)	(6.31)	(-2.07)			
27-30	5.77^{***}	19.38^{***}	-2.59	80.88^{***}	0.369	96
	(4.82)	(8.84)	(-1.53)			
30-33	6.26^{***}	19.38^{***}	-2.93	51.42^{***}	0.375	92
	(3.69)	(7.47)	(-1.22)			
33-36	6.77^{***}	15.72^{***}	-0.86	13.58^{**}	0.19	90
	(4.07)	(5.17)	(-0.28)			
Total	6.82	12.72	-1.98		0.22	1727

This table shows the results from the structural-break regressions for monthly realized kurtosis in the US. Regressions are run for each maturity individually and the row "Total" reports regression results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (14.04.2014) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to 12 lags (i.e. one year) using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to assess significance of the SupF-Test are taken from Bai and Perron (1998).

Table A.8: Structural break regression: EU realized kurtosis

Maturity (in months)	α	β_0	β_1	F	R^2	Observations
0-3	-	-	-	-	-	-
3-6	5.01^{***}	1.59	0.3	4.07	0.038	122
	(5.97)	(1.59)	(0.35)			
6-9	5.59^{***}	3.98^{***}	-0.38	22.89^{***}	0.103	143
	(4.89)	(2.73)	(-0.32)			
9-12	6.31^{***}	5.39^{***}	1.79	3.65	0.104	165
	(4.05)	(2.78)	(0.85)			
12-15	7.22^{***}	7.37^{***}	1.59	9.09^{*}	0.135	172
	(5.44)	(3.65)	(0.92)			
15-18	8.44^{***}	9.61^{***}	1.29	6.44	0.125	182
	(5.05)	(2.81)	(0.62)			
18-21	8.6^{***}	11.86^{***}	2.19	6.12	0.104	178
	(4.97)	(3.34)	(0.87)			
21-24	7.91^{***}	12.07^{***}	3.2^{*}	9.95^{**}	0.203	174
	(9.23)	(4.85)	(1.87)			
Total	7.23	5.81	1.74		0.06	1186

This table shows the results from the structural-break regressions for monthly realized kurtosis in the Eurozone. Regressions are run for each maturity individually and the row "Total" reports regression results from pooling all maturities together. The break-date parameter T_b is set to its least-square estimate (15.02.2015) uniformly for all regressions. The table displays the regression coefficients with t-statistics in parentheses, R^2s and the number of observations. Column F reports the statistic of the F-test run under the null of no structural change against the alternative hypothesis of a break at T_b. Error-covariances for computation of the t-statistics and F-tests are estimated accounting for heteroscedasticity and serial correlation up to 12 lags (i.e. one year) using the approach of Newey and West (1987). *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively. Critical values used to assess significance of the SupF-Test are taken from Bai and Perron (1998).